Sparsified State-Space Models are Efficient Highway Networks

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Abstract

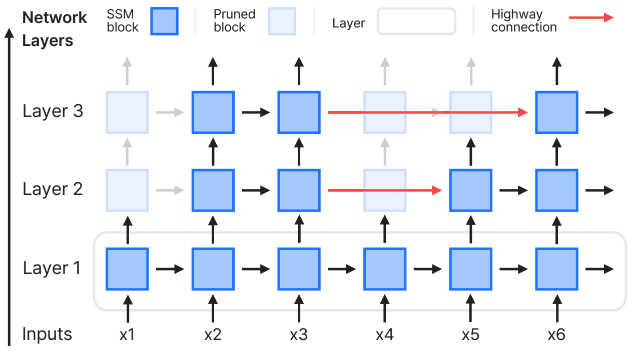
State-space models (SSMs) offer a promising architecture for sequence modeling, providing an alternative to Transformers by replacing expensive self-attention with linear recurrences. In this paper, we propose a simple yet effective trick to enhance SSMs within given computational budgets by sparsifying them. Our intuition is that tokens in SSMs are highly redundant due to gradual recurrent updates, and dense recurrence operations block the delivery of past information. In particular, we observe that upper layers of SSMs tend to be more redundant as they encode global information, while lower layers encode local information. Motivated by this, we introduce Simba, a hierarchical sparsification method for SSMs based on token pruning. Simba sparsifies upper layers more than lower layers, encouraging the upper layers to behave like highways. To achieve this, we propose a novel token pruning criterion for SSMs, measuring the global impact of tokens on the final output by accumulating local recurrences. We demonstrate that Simba outperforms the baseline model, Mamba, with the same FLOPS in various natural language tasks. Moreover, we illustrate the effect of highways, showing that Simba not only enhances efficiency but also improves the information flow across long sequences.

1 Introduction

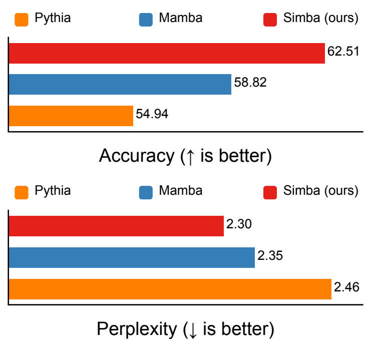
State-space models (SSMs) [1, 2] offer a promising architecture for sequence modeling, efficiently handling sequences using linear recurrence structures. Thanks to this efficiency, SSMs have shown potential as an alternative to Transformers [3], which use the self-attention mechanism, incurring high computational costs for long sequences. In particular, Mamba [2] has recently demonstrated that SSMs can scale up to billions of parameters and show comparable performance with Transformers in various domains, including language, image, and video [4–6].

After their success, numerous works have aimed to enhance SSMs and Mamba further. One popular approach involves hybrid models that combine Transformers and SSMs [7, 8]. This approach assists the models in retaining past information through the global memory of Transformers [9, 10]. However, this compromises the efficiency of SSMs by reintroducing expensive self-attention operations. Instead of sacrificing efficiency, we explore an alternative direction to improve SSMs by comparing models on a fixed computational budget. Specifically, we investigate the sparsification of large pre-trained SSMs, known to yield better models than training small ones from scratch [11].

To this end, we first analyze the behavior of tokens in pre-trained SSMs. We observe that tokens in SSMs are highly redundant, as they are gradually updated over sequences. This redundancy tends to be more severe in the upper layers, which encode global information, while lower layers encode local information. Furthermore, dense recurrence operations over the redundant tokens block the delivery of past information, potentially harming the contextual understanding of SSMs.



(a) Conceptual illustration of Simba



(b) Performance highlights

Figure 1: **Simba: hierarchical sparsification of SSMs via token pruning.** (a) We found that tokens in state-space models (SSMs) are highly redundant, especially in the upper layers. Motivated by this observation, we propose hierarchically sparsifying pre-trained SSMs by progressively pruning tokens across layers. This results in models with a trapezoidal shape, featuring sparse upper layers that act like highways, enhancing efficiency and information flow of the original SSM. (b) We highlight results comparing Simba-2.8b with Mamba and Pythia, all with the same number of FLOPS. We report the mean accuracy over 6 NLP benchmarks and perplexity on the PG-19 dataset with 2k context, following the setups in Section 4. Simba outperforms both models in accuracy and perplexity.

Inspired by this, we propose Simba,[1] a simple yet effective method to sparsify SSMs through token pruning (i.e., it is training-free). Our core idea is to sparsify SSMs into a hierarchical form, enforcing more sparsity in upper layers than in lower layers. As a result, the upper layers behave as highways to transmit past information, enhancing efficiency and facilitating the information flow across long sequences. Figure 1a illustrates the visual concept of our approach, obtaining a trapezoidal-shaped sparsified network.

To implement this, we propose a novel token pruning criterion for SSMs. Specifically, our score measures the global influence of each token on the final output by reformulating SSM equations to accumulate the effect from local recurrences. This approach can also be viewed as an SSM extension of attention-based token pruning criteria used for Transformers [12]. We found that our criterion outperforms intuitive baselines, such as uniform pruning of tokens with even intervals.

Our experiments show that Simba, obtained by sparsifying Mamba without any fine-tuning, significantly outperforms Mamba using the same number of FLOPS in various tasks. For instance, Simba consistently achieves better FLOPS-accuracy curves on 6 NLP benchmarks, including Lambada [13], HellaSwag [14], PIQA [15], ARC-Challenge [16], ARC-Easy [16], and WinoGrande [17]. Here, Simba obtained from Mamba-2.8b performs on par with the original Mamba-2.8b, despite using similar FLOPS to Mamba-1.4b, as highlighted in Figure 1b. For example, it achieves an average accuracy of 62.5% for 6 downstream NLP tasks, improving 58.8% of Mamba-1.4b.

We also demonstrate the language modeling ability of Simba by measuring perplexity on the PG-19 dataset [18] across different context lengths. Like the NLP benchmarks, Simba achieves better perplexity than Mamba using the same number of FLOPS. More importantly, Simba performs robustly over long sequences exceeding the pre-trained context length, such as twice longer than the trained length, unlike Mamba, which significantly deteriorates with length extrapolation. This supports the idea that the highway structures in Simba facilitate long sequence modeling.

We further investigate the effect of highways in Simba. Somewhat unexpectedly, we found that Simba performed better than its original unpruned Mamba in some of our experiments, potentially benefiting from the highways created at the upper layers. To further investigate the positive effect of highways, we examine the information flow across layers by assessing the influence of the sequence tokens on the final output. We observe that Mamba relies on tokens near the end across all layers, while Simba also focuses on earlier tokens at the upper layers, showcasing the role of highways.

[1]**S**pars**i**fied Ma**mba**, as we applied our method to Mamba; however, our principle can be generally applied to other SSMs.

2 Related work

State-space models (SSMs) are a powerful architecture for sequence modeling, integrating concepts from classic control theory [19] with recurrent neural networks [20]. The key idea of SSMs is to employ linear recurrence [1, 21–30], enabling efficient parallel inference and effective training, unlike Transformers using self-attention [3], which requires quadratic computation over the sequence length. As a result, SSMs have shown success in handling long sequences [31]. Recently, Mamba [2] further scaled up SSMs through a selection mechanism and hardware-aware algorithm, showing the potential of SSMs in challenging tasks such as language, audio, and video [4–6]. We aim to further improve Mamba through network sparsification, efficiently utilizing a fixed computational budget.

Sparsifying networks have been widely studied, primarily for training efficient models [32]. Most prior work focused on weight pruning, which removes unnecessary edges in weight matrices [33–36]. However, while weight pruning reduces model size, it does not enhance inference speed due to the batch computation nature of GPUs. To address this, structured pruning removes entire blocks at once, such as channels in CNNs [37] or attention heads in Transformers [38]. On the other hand, some works focused on the performance benefits of sparsified networks, suggesting that sparsifying large networks yields better models than training small models from scratch, known as lottery tickets [11, 39]. Our work relates to structured pruning, as removing tokens speeds up computation and reduces memory usage, and to lottery tickets, as it also improves performance.

Token pruning (or merging) is widely applied in Transformers to reduce heavy computation over long sequences [12, 40–44]. In particular, HOMER [45] has shown that hierarchical token pruning not only reduces computation but also enhances long context understanding by condensing global information into sparse tokens in the upper layers. Our work shares a similar spirit with HOMER but has notable differences. First, we explore token pruning for SSMs, unlike prior works focused on Transformers. To this end, we propose a novel token pruning criterion based on the global importance of tokens to the final output, derived from reformulating SSM equations to accumulate local recurrences. Second, by targeting SSMs, our hierarchical pruning scheme offers a novel interpretation of highway networks, connecting SSMs with classical recurrent networks with long-term memory. As a result, our token pruning approach enhances both the efficiency and information flow of SSMs.

Highway networks have been proposed to bypass information loss from dense computation through local residuals [46] or long skip connections [47–50]. Highways, also termed long-term memory, were used as a standard approach for sequence modeling before global computation methods like self-attention gained popularity [51–53]. However, integrating highways with SSMs proved challenging due to the linear recurrence structure of SSMs, not easily combined with skip connections. Instead of explicitly using such modules in SSM architectures, we introduce a simple and effective way to integrate highways by pruning dense token connections from the upper layers.

Tree RNNs have been explored for processing hierarchical data structures, such as word, sentence, and paragraph hierarchies in language [54–58]. Despite aligning with human intuition, most methods were unsuccessful due to complex architectures, while simple linear sequence modeling demonstrates its power [59]. Our token pruning naturally incorporates this hierarchical structure into SSMs, while favoring the success and scalability of linear sequence modeling.

3 Simba: Hierarchical sparsification for state-space models

In Section 3.1, we review the mathematical formula of state-space models (SSMs) and discuss observations on the token redundancy of Mamba. In Section 3.2, we describe our proposed hierarchical sparsification approach for SSMs and explain the token pruning criteria.

3.1 Motivation: Hierarchy in SSM token redundancy

Structured state-space model (S4) [1] is a family of recently proposed SSMs. In its continuous form, the SSM updates the state $h(t)$ using the input $x(t)$ and produces the output $y(t)$ according to Eq. (1) where A, B, and C refer to the parameters of SSMs. It discretizes the update rules for discrete sequences, as shown in Eq. (2).

$$h'(t) = Ah(t) + Bx(t), \quad y(t) = Ch(t) \tag{1}$$

$$h_t = \bar{A}h_{t-1} + \bar{B}x_t, \quad y_t = Ch_t \tag{2}$$

Algorithm 1 Practical implementation of Simba

```
procedure SIMBAFORWARD(input, num_keep)
    $A, B, C$, output $\leftarrow$ SSMForward($x$)                ▷ Forward and get SSM parameters
    score $\leftarrow$ GetScore($A, B, C$, input)                 ▷ Get influence score
    keep_indices $\leftarrow$ TopKIndex(score, num_keep)          ▷ Get indices of tokens to keep
    output $\leftarrow$ Gather(output, keep_indices)              ▷ Prune layer outputs
    return output
end procedure
```

Mamba [2] further improves this formulation using an input selectivity mechanism, creating the matrices $\bar{A}$, $\bar{B}$, and C dependent on the input x_t. Thus, the state update equation can be rewritten as follows, where $\bar{A}_t$, $\bar{B}_t$, and C_t denote the input-dependent matrices created using x_t.

$$h_t = \bar{A}_t h_{t-1} + \bar{B}_t x_t, \quad y_t = C_t h_t \tag{3}$$

Hierarchy in token redundancy. As the state update depends solely on the input and the immediate previous states, the model must compress all previous information into the state of each token. Thus, the states of SSM tokens are likely highly redundant, as states at similar positions would compress a similar set of information. To verify this, we visualize the token redundancy of Mamba in Figure 2 by measuring the cosine similarity between adjacent tokens across the network layers.

Our analysis indicates that tokens in the upper layers tend to exhibit more redundancy than those in the lower layers. One possible explanation is that SSMs process information in a hierarchical manner: lower layers focus on local information, while upper layers focus on global information. This aligns with findings from attention visualizations of Mamba models [60], where lower layers emphasize diagonal elements in the attention map, while upper layers highlight lower-triangular elements. Since upper layer states encapsulate global information, states at similar positions contain similar information, resulting in more redundant state representations.

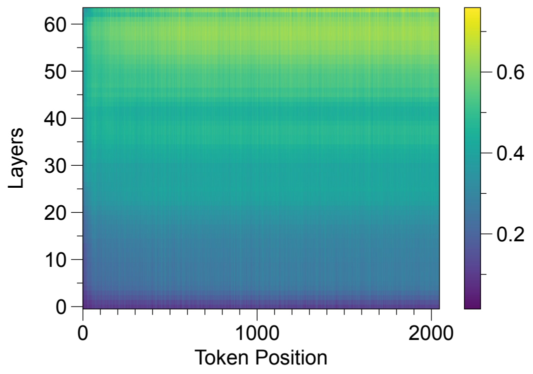


Figure 2: **Token redundancy of SSMs has a hierarchical structure.** We measure the cosine similarity between adjacent tokens of the Mamba-2.8b model across layers, averaged over documents from the PG-19 test dataset. The tokens are highly redundant, especially in the upper layers.

3.2 Hierarchical sparsification for SSMs

We introduce Simba, a simple yet effective sparsification approach that can be directly applied to any pre-trained state-space models (SSMs) in a plug-and-play manner. This introduces an efficient highway for effective sequential modeling. Based on prior motivation, Simba sparsifies the full SSM in a hierarchical manner: it sparsifies redundant tokens in upper layers while preserving the local information captured in lower layers, guided by our novel token importance criteria.

Hierarchical sparsification through token pruning. To achieve sparsity in SSMs in a hierarchical manner, we propose token pruning at each layer with a specified rate. Implementing this pruning technique at a particular layer automatically decreases the computational burden on subsequent layers, as pruned tokens are no longer propagated to the next layer. Consequently, by sequentially applying token pruning from lower to upper layers, the pruning ratio consistently increases during propagation, thereby introducing hierarchical sparsification to SSMs.

Token pruning criterion. We propose a novel token importance score for SSMs to prune the least important token from each layer. To achieve this, we leverage the influence function, which measures the change in the estimator when removing the target input. This calculation is straightforward due to the linear recurrence property of SSMs. Formally, for a given token sequence $(x_1, \cdots, x_T)$ of length

T and the final token output y_T of the layer, we estimate the influence of token x_t at position t by considering the updated output $y_T^{(t)}$ (obtained by removing x_t from y_T), as follows:

$$\begin{aligned}\Delta y_T(t) &:= y_T - y_T^{(t)} \\ &= \sum_{r=1}^{T} C_T \left(\prod_{k=r+1}^{T} \bar{A}_k \right) \bar{B}_r x_r - \sum_{r=1, r\neq t}^{T} C_T \left(\prod_{k=r+1}^{T} \bar{A}_k \right) \bar{B}_r x_r \\ &= C_T \left(\prod_{k=t+1}^{T} \bar{A}_k \right) \bar{B}_t x_t. \end{aligned} \tag{4}$$

Here, we found that the influence measure $\Delta y_T(t)$ aggregated with the max pooling (along with ℓ_2 norm) serves as an effective pruning criterion, denoted as $s(t) := \max(\Delta y_T(t))$; we use max pooling throughout the paper as it was slightly better than ℓ_2 norm. Based on the proposed score $s(t)$, we prune each layer by removing the tokens with the lowest scores to obtain a sparsified SSM.

Pruning schedule. The pruning schedule aims to balance the trade-off between efficiency and performance. Upper layers, with their ability to model global context, are better equipped to identify important tokens. Conversely, pruning tokens at earlier layers offers more significant computational savings. Therefore, we propose a linear pruning schedule, inspired by prior work on Transformers [42], where the number of active tokens is linearly reduced across all layers, resulting in a trapezoidal-shaped network after sparsification.

Highways in sparsified SSMs. Long-term dependency is a well-known challenge for recurrent models [51]. Dense recurrence operations tend to attenuate previous information, restricting the information flow across distant tokens. Our sparsification scheme addresses this issue by reducing the number of recurrence operations. As unimportant tokens are pruned during sparsification, upper layers can selectively focus on processing more important information without being burdened by dense recurrence operations on redundant tokens. This highway effect enables our pruning scheme to not only enhance efficiency but also facilitate the information flow across distant tokens (Section 4.3).

4 Experiments

In this section, we demonstrate the performance of Simba on diverse tasks. In Section 4.1, we evaluate its performance on 6 NLP benchmarks, consistently showing superior performance compared to dense models with equivalent computational resources. In Section 4.2, we assess the language modeling ability of Simba by measuring perplexity conditioned on various context lengths. In Section 4.3, we further investigate the highway effects of Simba. Finally in Section 4.4, we perform ablation studies on token pruning criteria and different pruning ratios, along with a simple fine-tuning experiment.

Common setups and baselines. We primarily apply our sparsification method to pre-trained Mamba models of various scales. For our method, we implement a linear pruning schedule that preserves 10% of the tokens at the final layer unless specified otherwise. The Simba models are generated by sparsifying Mamba models without any fine-tuning, following a plug-and-play approach. We conduct performance comparisons of Simba against Mamba [2] and Pythia [61] models that utilize a similar amount of computation. In NLP tasks consisting of a prompt and a label, we exclusively apply sparsification to the prompts to ensure an accurate evaluation of the label logits. The final token of the prompt remains unpruned, as its output is utilized for computing the label logits. Moreover, to accommodate the task structure, the token importance score is computed with respect to the final token of the prompt. For additional details, refer to Appendix A.

4.1 NLP benchmarks

In this section, we assess the language understanding capability of Simba by evaluating six downstream NLP tasks. Specifically, we present the performance and computational efficiency of Simba on the Lambada [13], HellaSwag [14], PIQA [15], ARC-Challenge [16], ARC-Easy [16], and Wino-Grande [17] benchmarks. Consistent with Mamba [2], we report accuracy normalized by sequence length for HellaSwag and ARC-Challenge, and accuracy for the other datasets. All evaluations use the LM evaluation harness from EleutherAI [62].

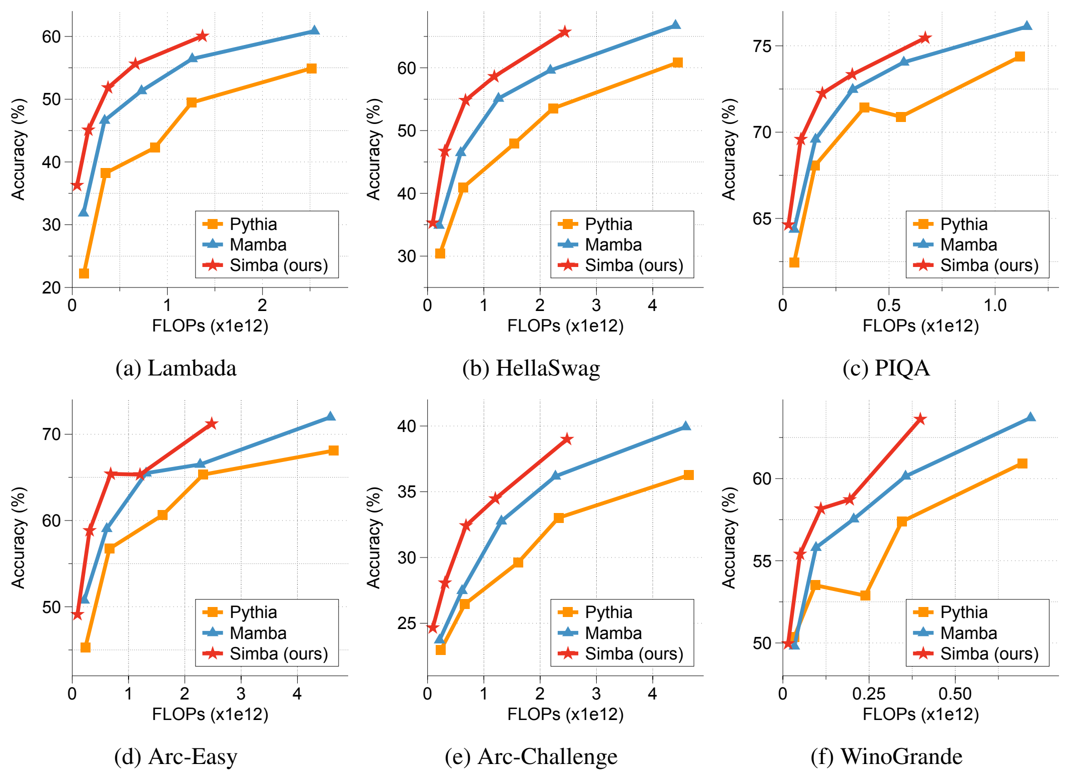


Figure 3: **Performance on NLP Benchmarks.** We visualize the FLOPs-accuracy curve of Mamba, Pythia, and Simba models of various scales on 6 NLP benchmarks. Across all benchmarks, Simba consistently outperforms the baselines using the same number of FLOPs.

We report evaluation accuracy and computational cost for each benchmark in Figure 3. As evident from the results, Simba provides the best accuracy-efficiency trade-off, consistently outperforming other models with the same number of FLOPs. This demonstrates that Simba can successfully make dense Mamba models sparse, advancing the frontier of the accuracy-efficiency trade-off. We provide the full results for all benchmarks in Table 4 of the Appendix.

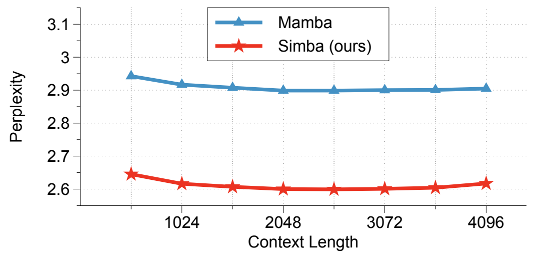
4.2 Language modeling

In this section, we evaluate the language modeling ability of Simba by measuring perplexity on long documents. Specifically, we measure the perplexity of short document snippets sampled from PG-19 dataset [18], conditioned on varying amounts of context. We keep the 100-token snippet fixed for all experiments to ensure that all perplexity measurements are done on the same set of tokens.

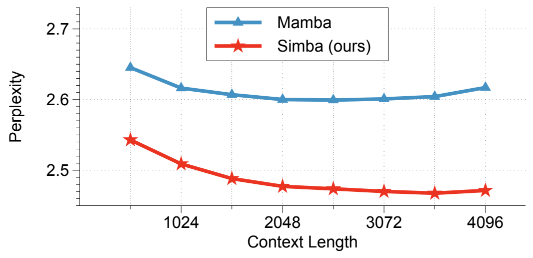
We report perplexity values conditioned on varying contexts in Figure 4. The results show that Simba consistently shows improved perplexity with similar computation, outperforming the dense Mamba models. We provide the full results, including Pythia, in Table 4 of the Appendix.

Long context capability. An intriguing observation is that Simba, unlike Mamba models, exhibits decreasing perplexity even after surpassing its pre-trained context limit of 2k tokens. Context limits, defining the maximum number of tokens a model can handle, often result in catastrophic performance drop if exceeded [45]. While SSMs are generally more resilient to this issue, the results indicate increasing perplexity for Mamba models with longer contexts, suggesting ineffective utilization of additional context. In contrast, Simba consistently demonstrates decreasing perplexity even with extended contexts, highlighting its adeptness at leveraging extra information.

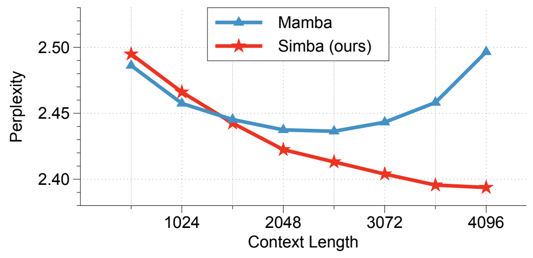
This benefit is likely attributed to the highways formed in the upper layers due to extensive sparsification. The performance drop with longer inputs typically results from a distribution shift in the training data, as the model never saw the long input length during training. For dense models like Mamba,



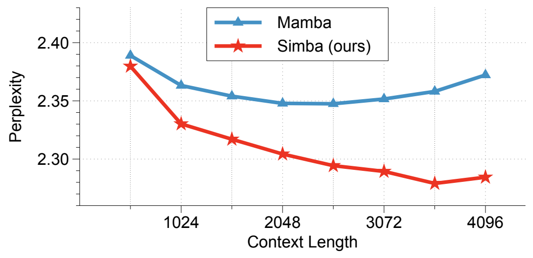
(a) Mamba-130m and Simba-370m



(b) Mamba-370m and Simba-790m



(c) Mamba-790m and Simba-1.4b



(d) Mamba-1.4b and Simba-2.8b

Figure 4: **Language modeling ability.** We measure the FLOPs-perplexity curves on the PG-19 test dataset. Simba models are compared against Mamba models that use similar computation. Simba not only outperforms Mamba with the same computation but also shows decreasing perplexity after its pre-trained context limit of 2k tokens.

this shift affects all network layers. Conversely, sparse models like Simba process significantly fewer tokens in the upper layers, thanks to extensive sparsification. Consequently, the upper layers remain unaffected by distribution shifts in input lengths, processing inputs more effectively.

In summary, our experiments demonstrate that Simba not only outperforms Mamba with the same computation but also exhibits superior long-context handling abilities, suggesting the benefits of highways.

4.3 Sparsified SSMs as highway networks

In this section, we further investigate the highway effects of Simba. First, we identify some scenarios where Simba models perform better than the original dense models despite using fewer FLOPs, potentially benefiting from the highways. Second, we examine the information flow in the model, showing that highways assist in obtaining information from earlier tokens, unlike dense SSMs, which over-rely on later tokens.

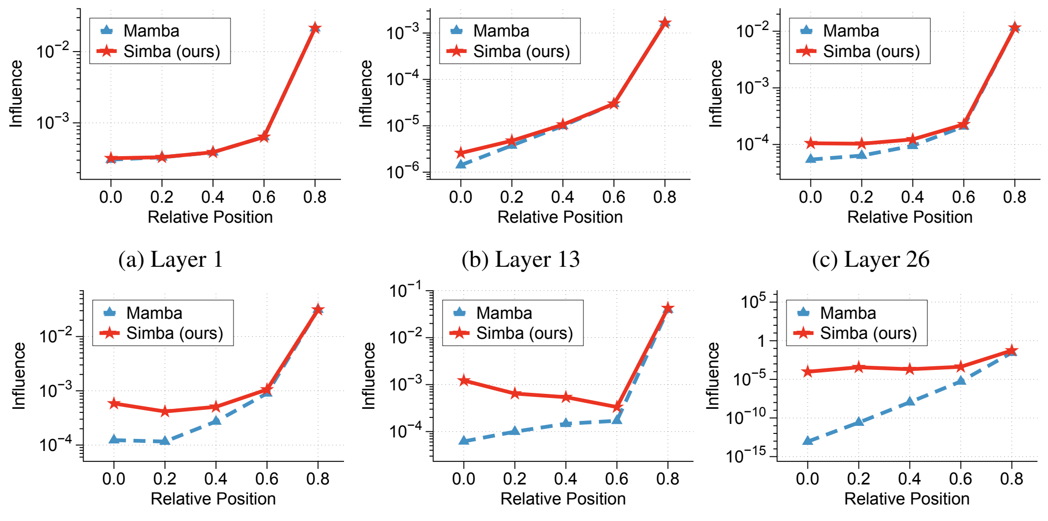
Comparison under same model sizes. In the previous experiments, we mainly compare models with the same number of FLOPS. Here, we provide an additional comparison between the dense and sparse models that have the same scale. Specifically, we evaluate Simba models with a more moderate pruning ratio, using a linear pruning schedule with 70% of tokens remaining at the final layer. We benchmark Mamba and Simba models using the 6 NLP benchmarks, following the setup in Section 4.1. We provide the evaluation results in Table 1.

Somewhat unexpectedly, we found that Simba models sometimes perform even better than the original model, indicating the possible benefits of highways created at the upper layers. The gain is more evident for smaller models, possibly because dense recurrence operations are more harmful to smaller state sizes, so highways provide more benefits.

Highways facilitate information flow from early tokens. We investigate how information flows through the dense and sparse SSM layers. Specifically, we measure the influence of tokens at each position on the final token, using our token importance score in Eq. (4) but normalized to equalize the contribution of each input document, i.e., $s(t)/||y_T||_2$. See Appendix A.2 for more details.

Table 1: **Comparsion between same model scales.** We compare Simba and Mamba on NLP benchmarks: Lambada (Lbd.), HellaSwag (HS), PIQA, Arc-Easy (Arc-E), Arc-Challenge (Arc-C), and WinoGrande (WG). We use a moderate pruning ratio for Simba, leaving 70% of the tokens at the final layer. Bold denotes the best results, showing that Simba often improves Mamba while using fewer FLOPS.

Model	Scale	Model Dim.	FLOPs (x1e12)	Lbd. acc. (↑)	HS acc. (↑)	PIQA acc. (↑)	Arc-E acc. (↑)	Arc-C acc. (↑)	WG acc. (↑)	Avg. acc. (↑)
Mamba	130m	768	0.48	31.84	34.88	64.36	**50.76**	23.72	49.80	42.56
Simba (ours)	130m	768	**0.39**	**32.43**	**35.05**	**64.42**	50.38	**24.32**	**49.88**	**42.75**
Mamba	370m	1024	1.38	46.61	46.46	69.59	59.05	27.47	**55.80**	50.83
Simba (ours)	370m	1024	**1.15**	**47.09**	**46.69**	**69.64**	**59.51**	**27.65**	55.49	**51.01**
Mamba	790m	1536	2.94	51.33	**55.12**	**72.47**	65.49	**32.76**	**57.54**	55.78
Simba (ours)	790m	1536	**2.47**	**51.82**	54.93	72.20	**65.87**	**32.76**	57.22	**55.80**



(a) Layer 1 (b) Layer 13 (c) Layer 26

(d) Layer 38 (e) Layer 51 (f) Layer 64

Figure 5: **Information flow across layers.** We visualize the information flow using the normalized token influence score. We compare Mamba-2.8b and Simba-2.8b, averaging scores over the PG-19 test dataset samples. The information flow of Simba flattens at the upper layers, indicating better information flow from early tokens.

We illustrate the results in Figure 5. Mamba demonstrates a consistent information flow pattern across all layers, with tokens near the end exerting more significant influence on the final token. Simba displays a similar trend in the lower layers. However, the slope flattens in the upper layers, indicating that they function as highways, facilitating the flow of information from early tokens.

4.4 Ablation study and analysis

This section presents ablation studies on token pruning criteria and different pruning ratios. We also perform a simple fine-tuning experiment to further improve the performance of Simba.

Pruning criteria. We compare our proposed pruning criteria in Eq. (4) with two baselines: "Random," which chooses tokens from random positions, and "Uniform," which chooses tokens from evenly distributed intervals. We visualize the efficiency-performance trade-off of Mamba-2.8b and Simba-2.8b models on the Arc-Challenge dataset in Figure 6. Random pruning significantly hurts performance, while Uniform pruning forms a strong baseline, highlighting the necessity of proper token selection. Simba further improves upon Uniform pruning by considering token influence.

Table 2: **Fine-tuning results.** We compare the language modeling perplexity on the test split of the PG-19 dataset, measured with different context lengths. The fine-tuned Simba model consistently outperforms its training-free counterpart.

Model	Scale	FLOPs	Within Context					Extrapolation		
		(x1e12)	0.5k	1k	1.5k	2k	2.5k	3k	3.5k	4k
Mamba	130m	0.48	2.943	2.917	2.907	2.899	2.899	2.900	2.901	2.905
Simba (training-free)	370m	0.68	<u>2.727</u>	<u>2.686</u>	<u>2.662</u>	<u>2.650</u>	<u>2.643</u>	<u>2.633</u>	<u>2.630</u>	<u>2.625</u>
Simba (fine-tuned)	370m	0.68	**2.723**	**2.678**	**2.658**	**2.645**	**2.637**	**2.630**	**2.626**	**2.621**

Pruning ratio. We compare the performance of Simba using different sparsity levels, with linear pruning schedules leaving 90%, 80%, 70%, 60%, 50%, 30%, and 10% of tokens at the final layer. Figure 6 presents the performance curves for different pruning criteria. Simba is robust to extreme sparsity, retaining performance even when pruning 90% of tokens at the final layer.

Fine-tuning. Although our method can be applied to pre-trained SSMs without training, we investigate if the performance can be improved with further fine-tuning. To this end, we perform a simple fine-tuning experiment, further training the Mamba-370m model with MiniPile dataset [63], which is a subset of the pre-training dataset (the Pile [64]) used for training Mamba. We provide the detailed training configurations in Appendix A.3.

Following the setup in Section 4.2, we evaluate the language modeling perplexity of 100 tokens using the PG-19 dataset, conditioned on varying amounts of context. We report the results in Table 2. For all context lengths, the fine-tuned Simba model consistently outperforms the training-free Simba model, suggesting that fine-tuning can further improve the performance of sparsified SSMs.

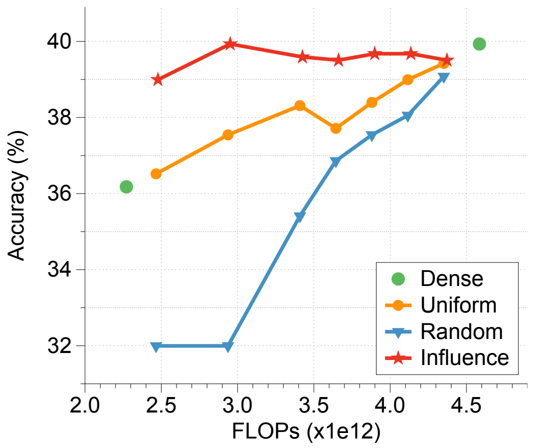


Figure 6: **Ablation study on pruning criteria.** Performance of sparsified Mamba-2.8b models on the Arc-Challenge dataset [16] evaluated using different pruning criteria across various pruning ratios. Our proposed token influence score in Eq. (4) performs the best, even remaining robust under severe sparsification. We also report the performance of the dense Mamba-1.4b and Mamba-2.8b models for comparison.

5 Conclusion

We propose Simba, which sparsifies pre-trained SSMs into a hierarchical form through token pruning. Simba outperforms Mamba with the same number of FLOPS in both accuracy on downstream NLP benchmarks and language modeling perplexity. Additionally, our pruning scheme creates highways in the upper layers, enhancing length extrapolation for long sequences and facilitating the information flow across distant tokens. We hope Simba inspires a broad community, including state-space models, sparse and efficient networks, and classic recurrent networks with highways.

Limitations. Our paper mainly focused on applying Simba to the pre-trained Mamba without adjustment. However, token pruning incurs distribution shifts from the original models, and further fine-tuning could reduce this misalignment. We demonstrate that simple fine-tuning can improve the performance of small models within a fixed computational budget in Section 4.4. However, a more sophisticated fine-tuning scheme tailored for sparse SSMs could be investigated.

Broader impacts. Our paper studies sequence models, with broad applications such as language, audio, and video generation. As our method enhances the efficiency and efficacy of these models, it holds the potential to impact a broader audience in generative AI. Hence, users of our method and sequence models should carefully read and follow the guidance from the community [65].

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A Detailed setups

We provide the detailed setups used in our experiments.

A.1 NLP benchmarks

Here, we provide the details for downstream experiments in Section 4.1. The six benchmarks were chosen according to the experiment setup of [2]. The metrics are also selected accordingly, where we measure answer perplexity and accuracy for Lambada, accuracy for PIQA, ARC-Easy, and Winogrande, and normalized accuracy for HellaSwag and Arc-Challenge. All measurements use the LM evaluation harness from EleutherAI [62].

Following the widely adopted practice [66], we evaluate the model's downstream performance using few-shot prompts. Following the setup in the open LLM leaderboard [66], we evaluate the model with 10-shot prompts for HellaSwag, 25-shot prompts for ARC-Easy and Arc-Challenge, and 5-shot prompts for WinoGrande. For benchmarks not considered in the leaderboard, we evaluate the models with 5-shot prompts.

For the visualization in Figure 3, we measure FLOPs separately for each benchmark and each model. First, we measure each benchmark's mean prompt and answer length, as shown in Table 3. Then, we measure FLOPs by forwarding an input that matches the mean lengths.

Table 3: **Downstream task details.** We provide the details of our downstream evaluations in Section 4.1, including the number of few-shot prompts, average prompt lengths, average answer lengths, and metrics used for evaluation.

Dataset	Few-shot Prompts	Prompt Length (avg.)	Answer Length (avg.)	Metrics
Lambada [13]	5	507.44	1.47	ppl./ acc.
HellaSwag [14]	10	877.25	29.86	acc_norm
PIQA [15]	5	229.53	22.82	acc.
ARC-Easy [16]	25	913.81	5.00	acc.
ARC-Challenge [16]	25	913.81	5.00	acc_norm
WinoGrande [17]	5	143.17	5.67	acc.

A.2 Information flow visualization

Here, we provide the details for our information flow visualization experiments in Figure 5. We measure the normalized token influence score $s(t)/||y_T||_2$ across all documents from the PG-19 test set, truncated at 1000 tokens. For all samples, we gather the influence score into five bins according to the position of the tokens. We report the average influence scores for each bin.

A.3 Fine-tuning with Simba

We provide the details for our fine-tuning experiments in Section 4.4.

Loss design. The language modeling loss cannot be directly applied to sparsified models because the output logit shape does not match the label shape due to pruning. We apply the language modeling loss only to the available output logits, which are trained to predict the next token in the input sequence. We also add the standard language modeling loss computed using dense forwarding for stable training.

Training. We train the model for 400 steps on the MiniPile dataset [63], a subset of the Pile dataset [64] with similar data distribution. We use AdamW optimizer with a learning rate of 5e-5. We schedule the learning rate with a linear warmup for 10% of the total training steps and cosine learning rate decay for the remaining steps. We randomly select the pruning ratio between 0% and 90% for each sample.

B Additional results

The tables below present the full values reported in our experiments.

B.1 Detailed results for downstream evaluations

Table 4: The full results table corresponding to Figure 3. Bold denotes the best results.

Model	Scale	FLOPs (x1e12)	Lbd. perp. (↓)	Lbd. acc. (↑)	HS acc. (↑)	PIQA acc. (↑)	Arc-E acc. (↑)	Arc-C acc. (↑)	WG acc. (↑)	Avg. acc. (↑)
Simba (ours)	130m	0.23	31.63	36.28	35.29	64.64	49.12	24.66	49.96	43.32
Pythia	160m	0.66	$> 10^2$	22.21	30.42	62.46	45.29	22.95	50.36	38.95
Mamba	130m	0.53	35.81	31.84	34.88	64.36	50.76	23.72	49.80	42.56
Simba (ours)	370m	0.75	**16.79**	**45.10**	**46.72**	**69.59**	**58.84**	**28.07**	**55.41**	**50.62**
Pythia	410m	1.86	25.65	38.22	40.95	68.06	56.73	26.45	53.51	47.32
Mamba	370m	1.51	12.34	46.61	46.46	69.59	59.05	27.47	55.80	50.83
Simba (ours)	790m	1.67	**10.10**	**51.84**	**54.82**	**72.25**	**65.40**	**32.42**	**58.17**	**55.82**
Pythia	1b	4.27	16.01	42.27	47.92	71.44	60.65	29.61	52.88	50.80
Mamba	790m	3.24	9.44	51.33	55.12	72.47	**65.49**	32.76	57.54	55.78
Simba (ours)	1.4b	2.95	**8.41**	**55.61**	**58.61**	**73.34**	65.32	**34.47**	58.72	**57.68**
Pythia	1.4b	6.19	10.35	49.46	53.54	70.89	65.32	33.02	57.38	54.94
Mamba	1.4b	5.60	7.31	56.43	59.60	74.05	66.50	36.18	60.14	58.82
Simba (ours)	2.8b	6.07	**6.29**	**60.06**	**65.70**	**75.46**	**71.21**	**38.99**	**63.61**	**62.51**
Pythia	2.8b	12.21	7.69	54.93	60.85	74.37	68.10	36.26	60.93	59.24
Mamba	2.8b	11.31	5.80	60.85	66.73	76.12	71.97	39.93	63.69	63.21

B.2 Detailed results for perplexity evaluations

Table 5: The full results table corresponding to Figure 4. Bold denotes the best results.

Model	Scale	FLOPs	Within Context				Extrapolation		
		(x1e12)	0.5k	1k	1.5k	2k	2.5k	3k	3.5k	4k
Simba (ours)	130m	0.21	3.060	3.006	2.986	2.974	2.961	2.956	2.948	2.942
Pythia	160m	0.60	3.166	3.134	3.128	3.120	3.195	6.333	7.883	7.986
Mamba	130m	0.48	2.943	2.917	2.907	2.899	2.899	2.900	2.901	2.905
Simba (ours)	370m	0.68	**2.727**	**2.686**	**2.662**	**2.650**	**2.643**	**2.633**	**2.630**	**2.625**
Pythia	410m	1.69	2.750	2.713	2.702	2.691	2.923	7.611	8.918	9.314
Mamba	370m	1.38	2.645	2.616	2.607	2.600	2.599	2.601	2.604	2.617
Simba (ours)	790m	1.52	**2.543**	**2.509**	**2.488**	**2.477**	**2.474**	**2.470**	**2.468**	**2.471**
Pythia	1b	3.88	2.607	2.573	2.560	2.559	5.665	6.582	6.817	7.003
Mamba	790m	2.94	**2.486**	**2.457**	2.445	2.437	2.436	2.443	2.458	2.497
Simba (ours)	1.4b	2.68	2.495	2.466	**2.443**	**2.422**	**2.413**	**2.404**	**2.396**	**2.394**
Pythia	1.4b	5.63	2.513	2.474	2.465	2.456	3.003	6.696	7.321	7.775
Mamba	1.4b	5.09	2.389	2.363	2.354	2.348	2.347	2.352	2.358	2.372
Simba (ours)	2.8b	5.52	**2.380**	**2.330**	**2.317**	**2.304**	**2.294**	**2.289**	**2.279**	**2.284**
Pythia	2.8b	11.11	2.382	2.345	2.332	2.326	6.689	7.275	7.266	7.244
Mamba	2.8b	10.28	2.281	2.254	2.242	2.235	2.236	2.254	2.314	2.547

C Additional information

C.1 Compute resources

All experiments are done on RTX-3090 or RTX-2080 GPUs. All FLOPs reported in the paper indicate the computation required for running a single forward pass, so the number of data samples must be multiplied to calculate the amount of computation required for the full experiments. We further acknowledge that the full research project required more computing than the experiments reported in the paper, including the preliminary experiments and failed experiments.

C.2 License for existing assets

Here, we provide the license for all datasets and models used in our experiments. Apache 2.0 license is applied for the pretrained Mamba models, Pythia models, PG-19 dataset, and WinoGrande dataset. CC BY 4.0 license is applied for the Lambada dataset. CC BY-SA 4.0 license is applied for ARC-Challenge and ARC-Easy datasets. MIT license is applied for the PIQA dataset, HellaSwag dataset, and MiniPile dataset.