ThinK: Thinner Key Cache by Query-Driven Pruning

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Abstract

Large Language Models (LLMs) have revolutionized the field of natural language processing, achieving unprecedented performance across a variety of applications. However, their increased computational and memory demands present significant challenges, especially when handling long sequences. This paper focuses on the long-context scenario, addressing the inefficiencies in KV cache memory consumption during inference. Unlike existing approaches that optimize the memory based on the sequence length, we identify substantial redundancy in the channel dimension of the KV cache, as indicated by an uneven magnitude distribution and a low-rank structure in the attention weights. In response, we propose THINK, a novel query-dependent KV cache pruning method designed to minimize attention weight loss while selectively pruning the least significant channels. Our approach not only maintains or enhances model accuracy but also achieves a reduction in KV cache memory costs by over 20% compared with vanilla KV cache eviction and quantization methods. For instance, THINK integrated with KIVI can achieve a $2.8\times$ reduction in peak memory usage while maintaining nearly the same quality, enabling up to a $5\times$ increase in batch size when using a single GPU. Extensive evaluations on the LLaMA and Mistral models across various long-sequence datasets verified the efficiency of THINK, establishing a new baseline algorithm for efficient LLM deployment without compromising performance.

1 Introduction

Large language models (LLMs) [17, 5, 36, 42, 43, 38, 37] have emerged as a dominant paradigm in natural language processing, achieving state-of-the-art performance across various tasks. A key principle, the Scaling Law [25], suggests that LLMs exhibit emergent abilities as model size increases, improving their capacity to understand complex context and handle long sequences [52]. This growth in capacity enables LLMs to generate coherent, contextually accurate responses and supports a variety of downstream applications, such as document summarization [56, 55], code generation [7], solving mathematical problems [18, 59, 46, 29], and conversational AI [35, 36].

Despite their success in various applications, generating outputs with LLMs incurs significant computational and financial costs, which rise with increasing model size and sequence length. Both the training [40, 19, 12] and inference [1] stages involve frequent generation, further contributing to these costs. Consequently, efficient LLMs have gained traction in recent years [21, 45]. To address these challenges, quantization [15, 30, 10, 53] and pruning methods [41, 14] are employed to reduce model size. Additionally, the key-value (KV) cache, stored in GPU memory alongside model parameters, scales linearly with both sequence length and batch size, creating a substantial memory burden when handling long sequences. Consequently, effective management of extended contexts is essential for the practical deployment of LLMs. In this paper, we focus on the long-context scenario, aiming to reduce memory consumption associated with processing lengthy sequences.

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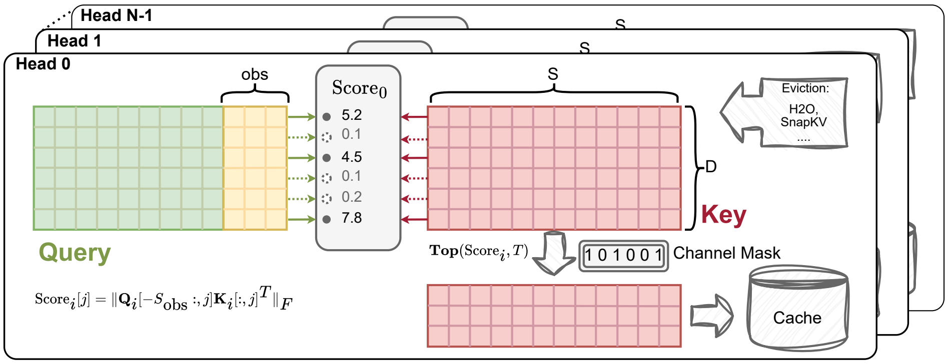

Figure 1: An illustration of the pruning procedure of THINK. Within each head, scores are calculated for each channel, and only the top T channels out of D are selected for retention. A binary channel mask, along with the pruned keys, is subsequently stored in the cache memory.

Specifically, the number of KV cache parameters is the product of batch size B, sequence length S, number of layers L, number of heads N, channel size per head D, *i.e.*, $\mathbf{K}, \mathbf{V} \in \mathbb{R}^{B\times S\times L\times N\times D}$, which need to be stored in the GPU memory during inference. To reduce memory and computational costs during inference, efficiency can only be achieved by pruning the dimensions across S, L, N, D or applying quantization to the caches. It is well-acknowledged that token importance tends to be sparse. Consequently, KV eviction algorithms have been proposed to reduce the memory footprint by addressing the sequence length dimension S [51, 28, 58, 27]. Additionally, inter-layer redundancy has been explored [31, 49, 4] to address the layer dimension L. Despite these advances, existing methods have largely overlooked the channel dimension D. In this paper, we highlight that the magnitudes across key cache channel dimensions are significantly imbalanced, and we observe a low-rank structure in attention weights. Based on these findings, we hypothesize that the channel dimension of the key cache exhibits redundancy. Consequently, we focus on exploring the redundancy in the KV cache along dimension D, aiming to develop strategies that reduce memory costs without compromising performance.

In this paper, we introduce THINK, a simple yet effective method for KV cache pruning. To pinpoint the least significant channels, we formulate the problem as an optimization task, aiming to minimize the loss in attention weights caused by pruning. To effectively address this problem, we propose a novel query-dependent criterion that assesses the importance of each channel. Using this criterion, we then select the most critical channels in a greedy fashion. We evaluate THINK using the LLaMA [34] and Mistral [23] models, and validate its effectiveness across various long-sequence datasets. The results indicate that, when paired with token eviction and KV cache quantization methods, THINK not only maintains comparable or superior accuracy but also reduces KV cache memory costs by over 20%.

Contributions. This work pioneers the investigation into the sparsity within the channels of the KV cache. Specifically, we uncover that the activated key cache is sparse for a given query. This insight allows us to prune the key cache channels using a query-induced norm. Building on this insight, we introduce THINK, the first channel pruning method specifically designed for KV cache. THINK reduces the dimensionality of the cache channels, leading to linear savings in memory usage. As a plug-and-play technique, THINK is orthogonal to other KV cache compression schemes (*e.g.* KV cache eviction, quantization). Our extensive experiments demonstrate THINK's remarkable efficiency on the LLaMA and Mistral models. Moreover, we explore the potential extension of THINK to value cache pruning (THINKV), showcasing the broad applicability of our method.

2 Observations

We identify several key observations (in Appendix A.1) that motivate our approach to pruning the channels of the KV cache. Specifically, we visualize the magnitude of the KV cache and perform singular value decomposition (SVD) on the attention mechanism of the LLaMA model.

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Notations. We use uppercase letters (*e.g.*, X, Y) to denote scalar values and boldface uppercase letters (*e.g.*, $\mathbf{Q}, \mathbf{K}$) to denote matrices and tensors. The notation $\|\cdot\|_p$ denotes the l_p-norm for vectors. Unless otherwise specified, $\|\cdot\|$ denotes the l_2-norm. The Frobenius norm is denoted by $\|\cdot\|_F$. The floor function is denoted by $\lfloor\cdot\rfloor$, and the ceiling function is denoted by $\lceil\cdot\rceil$.

3.1 Preliminary Study of KV Cache Optimization

In scenarios with extended contexts or batch processing, the main limitations in terms of memory and speed are due to the handling of the KV cache size. Considering a batch of requests to a Large Language Model (LLM) service that provides a long input prompt consisting of tokens $[x_{B1}, ..., x_{BS}]$, the total KV cache size can be computed as follows: $2 \times B \times S \times L \times N \times D$, where L is the number of layers, N is the number of heads, D is the head dimension. The KV cach size grows linearly as the batch size B and sequence length S. For a model with multihead attention (MHA) [44], such as LLaMA2-7B [43], a context length of 2048 and a batch size of 13 require storing a 13 GB KV cache, which is equivalent to the size of the model parameters. The KV cache must be transferred from off-chip memory (HBM) [22] to on-chip memory (cache) for each token generated, leading to a memory bottleneck. This substantial memory demand highlights the challenges in managing large-scale models and the need for efficient memory utilization strategies. Current methods optimize the KV cache based on the sequence length S [51, 58, 28] and precision [20, 32]. We will introduce a new method, THINK, to optimize it from the perspective of the number of head dimensions D.

Table 1: Performance comparison of pruning key cache by l_p norm on LongBench.

Method		Single-Document QA			Multi-Document QA			Summarization			Few-shot Learning			Synthetic		Code		Avg.
		NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
H2O	0.0	23.52	17.93	34.68	42.11	33.52	19.92	22.11	22.56	23.82	41.00	90.46	40.20	5.87	69.50	56.71	51.69	37.23
$+l_1$	0.3	23.38	17.15	34.99	40.56	31.49	19.90	21.37	22.13	23.44	40.50	90.10	40.65	5.41	69.00	58.64	54.99	37.11
$+l_1$	0.4	23.51	15.40	34.37	40.71	31.28	20.24	21.25	22.29	22.54	38.50	89.22	39.27	5.87	68.33	58.47	54.33	36.60
$+l_2$	0.3	23.98	17.04	35.19	39.27	31.29	20.40	21.62	22.46	23.34	40.50	89.75	40.71	5.54	68.67	60.12	58.52	37.40
$+l_2$	0.4	23.76	16.23	32.19	40.23	32.13	20.69	21.30	22.25	23.20	39.50	89.61	40.24	5.66	69.00	60.09	59.45	37.22
SnapKV	0.0	24.84	23.96	38.77	42.75	34.55	20.87	22.26	22.61	23.97	70.00	90.52	40.29	5.81	69.50	59.04	51.81	40.10
$+l_1$	0.3	24.43	24.63	40.11	41.83	33.47	21.22	21.47	22.41	23.73	66.50	90.39	40.20	5.70	68.10	61.04	55.37	40.04
$+l_1$	0.4	24.58	24.87	39.30	42.76	31.95	20.47	20.95	22.22	23.42	55.50	90.22	39.13	5.82	68.39	60.71	56.10	39.15
$+l_2$	0.3	24.47	24.73	38.16	41.86	32.23	20.23	21.59	22.45	23.77	67.50	90.33	40.31	5.70	68.42	62.65	60.07	40.28
$+l_2$	0.4	24.52	23.75	38.35	42.42	32.96	20.39	21.21	22.28	23.41	60.00	90.20	39.59	5.75	68.29	61.96	60.59	39.74

Magnitude based Pruning: Based on the observations in Figure 3 which depicts the significant variation in the magnitudes across different channels, one straightforward criterion is to use the norm of the magnitude to measure the importance of different channels in key cache.

$$M_{n,d} = \left\| K[n, :, d] \right\|_p. \tag{1}$$

Given pruning ratio λ, We only keep $T = \lfloor(1 - \lambda)D\rfloor$ most important channels among the D channels of each head: $I = \mathbf{Top}_T(M, T)$ where $\|\cdot\|_p$ is the l_p norm of each channel. $n \in [1, N]$ and $d \in [1, D]$ are indicators of heads and channels in key cache. $I \in (\mathbb{Z}^+)^{N \times T}$ stores the indicators of the top T values in tensor M per head.

In Table 1, we present the results of key cache pruning with various pruning ratios applied to the LLaMA-3-8B model. We utilize the l_1 and l_2 norms as criteria for evaluation, and validate performance using the LongBench benchmark [2]. Compared to the baseline methods, H2O [58] and SnapKV [28], both with a KV length of 512, we further prune the channels of the key cache. A 30% pruning ratio can maintain accuracy; however, increasing it to 40% results in significant performance degradation, especially for l_1 norm based pruning. The results of magnitude-based pruning support our assumption that the key cache is redundant in the channel dimension. These results also indicate the need for a better pruning matrix to achieve higher pruning ratios effectively.

3.2 Query-Driven Pruning

For each head, the attention scores are computed using the queries and keys, and then applied to the values. The formula for the attention for head i is: Attention$(\mathbf{Q}_i, \mathbf{K}_i, \mathbf{V}_i) = \text{softmax}(\frac{\mathbf{Q}_i\mathbf{K}_i^T}{\sqrt{D}})\mathbf{V}_i$, where $\mathbf{Q}_i, \mathbf{K}_i, \mathbf{V}_i \in \mathbb{R}^{S \times D}$. When one channel of $\mathbf{K}_i$ is pruned, the corresponding channel in $\mathbf{Q}_i$ will also be removed. We aim to find the optimal subset of channels to prune, denoted by the selection matrix $\mathbf{S} \in \{0,1\}^{D \times D}$, where $\mathbf{S}$ is a diagonal matrix with binary entries (1 for keeping a channel, 0 for pruning it). To better maintain the performance after pruning the channels, we minimize the Frobenius norm of the difference between the original and pruned attention weights: $\min_{\mathbf{S}} \|\mathbf{Q}_i\mathbf{K}_i^T - \mathbf{Q}_i\mathbf{S}(\mathbf{K}_i\mathbf{S})^T\|_F$. Given a pruning ratio λ, it can further expanded as:

$$\begin{aligned} \min_{\mathbf{S}} \quad & \left\|\mathbf{Q}_i\mathbf{K}_i^T - \mathbf{Q}_i\mathbf{S}\mathbf{K}_i^T\right\|_F \\ \text{subject to} \quad & \text{trace}(\mathbf{S}) = \lfloor (1-\lambda)D \rfloor \\ & \mathbf{S} = \text{diag}(s_1, s_2, \ldots, s_D), \text{ where } s_j \in \{0,1\} \end{aligned} \tag{2}$$

For simplicity, we use greedy algorithm to optimize $\mathbf{S}$. To achieve the pruning goal, we define a criterion for evaluating the importance of each channel and greedily select the channels with largest scores: $\text{Score}_i[j] = \left\|\mathbf{Q}_i[:,j]\mathbf{K}_i[:,j]^T\right\|_F, \quad I_i = \mathbf{Top}_T(\text{Score}_i, T)$. Here's a detailed explanation of why it optimizes the selection matrix. The $\text{score}_i[j]$ measures the magnitude of the interaction between the query and key vectors for channel j in each head i. By selecting channels with the highest interaction magnitudes, we aim to retain the most significant contributions to the attention mechanism. This criterion ensures that the selected channels preserve the primary information flow in the attention computation, thereby minimizing the loss of important information.

Observation Window. Based on the observations in SnapKV [28] that the last window of input sequence recognizes highly similar attention pattern with generation. To reduce the computation cost, we only use the last S_{obs} window to calculate the score: $\|\mathbf{Q}_i[-S_{\text{obs}} :, j]\mathbf{K}_i[:, j]^T\|_F$.

4 Experiment Results

4.1 Settings

Benchmark Datasets. We evaluate our proposed method against state-of-the-art KV cache compression methods on two widely recognized benchmarks: LongBench and Needle-in-a-Haystack. *LongBench* [2] is designed to comprehensively assess the long context understanding capabilities of LLMs. It includes 17 datasets covering six different tasks: single-document QA, multi-document QA, summarization, few-shot learning, synthetic tasks, and code completion. The average input length of LongBench is 6,711 words, which necessitates reducing the KV cache to lower memory usage for inference. *Needle-in-a-Haystack* [24] is a recently developed benchmark that tests a model's ability to accurately locate a small but crucial piece of information (the "needle") embedded within a lengthy document (the "haystack"). The random positioning of the needle in this challenge serves as a critical test to determine whether KV cache compression methods can retain essential information without loss of accuracy.

Baseline Approaches. The baseline methods in our evaluations include Heavy Hitter Oracle (H2O), SnapKV and KIVI, all of which are the state-of-the-art KV cache compression methods but use different strategies. **H2O** [58] is designed to reduce memory usage by dynamically managing the balance between recent tokens and Heavy Hitter (H2) tokens. H2 tokens represent a small set of tokens that contribute most of the value when computing attention scores. **SnapKV** [28] introduces an automated compression mechanism that selects clustered, important KV positions for each attention head, optimizing the KV cache without sacrificing performance. **KIVI** [32] reduces memory overhead by quantizing the KV cache into lower-precision formats, significantly lowering the memory cost while preserving model accuracy.

Implementation Details. In this paper, we use LLaMA-2-7B-chat, LLaMA-3-8B-Instruct, LLaMA-3-70B-Instruct [34] and Mistral-7B-Instruct-v0.2 [23] as the backbone LLMs, both accessible via HuggingFace [48]. Our THINK aims to prune channels of the key cache, which is agnostic to KV cache compression methods. If there is no other statement, we prune the key cache by default. All the experiments are conducted using NVIDIA A100 GPUs. To ensure a fair comparison between KV cache compression methods and their integration with THINK, we applied consistent hyperparameters across both settings. For instance, when comparing SnapKV and SnapKV integrated with THINK, we used a maximum pooling kernel size of 7 and an observation window size of 32, maintaining the same KV-size for both configurations.

Table 2: Performance comparison of key cache pruning on LLaMA-3-(8B/70B)-Instruct on LongBench. THINK (λ) indicates we prune the key cache channels with a pruning ratio of λ.

Method	Single-Document QA			Multi-Document QA			Summarization			Few-shot Learning			Synthetic		Code		Avg.
	NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
							LLaMA-3-8B-Instruct, KV-size Full										
ALL KV	25.56	32.27	39.71	43.56	35.09	21.18	28.71	23.26	26.64	73.50	90.48	42.33	4.80	69.25	59.29	54.05	41.86
							LLaMA-3-8B-Instruct, KV-size 128										
H2O	22.12	13.20	31.61	37.79	32.71	18.45	20.32	22.02	21.10	38.50	87.75	39.14	5.83	69.50	55.06	50.97	35.38
+THINK (0.4)	22.85	14.55	29.49	38.63	30.84	18.90	20.12	21.96	20.68	38.50	86.38	38.40	5.50	69.17	57.93	56.12	**35.63**
+THINK (0.5)	23.47	14.06	28.67	38.35	30.21	17.87	19.69	21.94	19.95	38.50	87.14	38.07	4.92	69.50	57.99	56.66	35.44
SnapKV	21.19	13.55	32.64	38.75	29.64	18.73	18.98	21.62	20.26	45.00	88.36	37.64	5.13	68.85	55.84	51.82	35.50
+THINK (0.4)	22.11	14.67	32.49	36.25	28.63	18.80	18.93	21.49	20.14	44.50	88.11	38.32	5.75	69.17	58.21	55.89	**35.84**
+THINK (0.5)	21.79	14.73	32.03	37.52	27.86	18.28	18.50	21.52	19.71	43.50	86.00	38.35	5.59	69.50	57.96	56.96	35.61
							LLaMA-3-8B-Instruct, KV-size 512										
H2O	23.52	17.93	34.68	42.11	33.52	19.92	22.11	22.56	23.82	41.00	90.46	40.20	5.87	69.50	56.71	51.69	37.23
+THINK (0.4)	23.76	17.80	33.80	40.39	30.70	19.09	21.82	22.51	23.78	41.00	90.16	40.67	5.15	69.25	60.77	57.58	**37.39**
+THINK (0.5)	24.17	16.96	35.76	39.47	30.29	18.67	21.39	22.59	23.06	41.00	89.81	40.35	5.23	69.33	60.20	58.34	37.29
+THINK (0.6)	23.40	14.83	32.62	38.47	30.97	19.81	20.80	22.04	21.60	40.00	88.79	38.90	5.36	69.50	58.28	57.65	36.44
SnapKV	24.84	23.96	38.77	42.75	34.55	20.87	22.26	22.61	23.97	70.00	90.52	40.29	5.81	69.50	59.04	51.81	40.10
+THINK (0.4)	24.58	25.44	37.03	41.87	33.45	20.58	21.77	22.42	24.16	70.00	90.39	40.29	6.06	69.50	62.05	59.23	**40.55**
+THINK (0.5)	24.85	25.10	37.06	41.58	32.34	20.60	21.61	22.44	23.66	69.50	90.39	39.70	5.84	69.79	61.57	59.42	40.34
+THINK (0.6)	25.98	22.77	38.37	40.44	33.19	19.90	20.84	22.21	22.55	59.00	90.32	38.12	6.39	69.50	59.14	58.40	39.20
							LLaMA-3-8B-Instruct, KV-size 1024										
H2O	25.62	22.16	36.81	41.01	33.53	19.41	23.28	22.65	25.41	46.50	90.82	41.78	5.79	69.25	59.69	55.50	38.70
+THINK (0.4)	25.52	21.93	37.17	41.56	31.22	20.17	22.89	22.95	25.21	47.00	90.74	41.34	5.57	69.50	62.58	58.67	**39.00**
+THINK (0.5)	25.41	22.19	37.64	40.92	31.27	18.66	22.17	22.22	24.84	46.50	90.34	40.59	5.20	69.50	61.71	57.99	38.57
+THINK (0.6)	24.06	17.80	37.85	38.63	29.98	19.40	21.41	22.32	23.42	44.50	90.16	39.43	5.84	69.50	58.31	58.73	37.58
SnapKV	24.62	25.99	37.64	43.84	34.99	20.00	24.28	22.39	25.63	72.5	90.56	40.41	5.36	69.25	60.57	56.11	40.88
+THINK (0.4)	24.88	27.72	38.60	43.16	32.44	20.67	24.21	22.79	25.56	71.50	90.45	40.94	5.93	69.50	62.77	59.45	**41.29**
+THINK (0.5)	24.82	27.26	39.66	42.82	32.09	19.56	23.52	22.48	25.34	71.50	90.43	40.74	5.20	69.50	62.46	59.75	41.07
+THINK (0.6)	24.46	27.35	38.22	41.96	31.64	20.18	21.89	22.83	23.68	70.00	90.19	38.69	6.10	69.50	58.87	59.26	40.30
							LLaMA-3-8B-Instruct, KV-size 2048										
H2O	25.56	26.85	39.54	44.30	32.92	21.09	24.68	23.01	26.16	53.00	90.65	41.84	4.91	69.25	58.43	51.31	39.59
+THINK (0.4)	25.56	26.31	39.20	42.96	31.81	20.53	24.23	23.35	25.90	53.50	90.56	41.03	5.52	69.25	62.10	59.00	**40.05**
+THINK (0.5)	25.01	25.37	38.82	42.32	31.27	20.50	23.78	23.21	26.03	53.00	90.37	40.86	5.13	69.50	61.91	58.95	39.75
+THINK (0.6)	24.37	22.14	37.77	40.13	29.50	20.26	22.09	22.76	24.78	49.50	90.16	39.69	5.56	69.50	59.24	58.78	38.51
SnapKV	25.86	29.55	41.10	44.99	35.80	21.81	25.98	23.40	26.46	73.50	90.56	41.66	5.17	69.25	58.67	51.52	41.58
+THINK (0.4)	25.41	29.79	39.21	43.35	33.96	21.49	25.78	23.11	26.23	73.00	90.56	41.79	5.81	69.50	62.45	59.19	**41.91**
+THINK (0.5)	25.00	30.25	39.27	43.23	32.93	21.24	25.16	23.01	26.5	73.00	90.37	41.26	5.45	69.50	62.3	59.84	41.77
+THINK (0.6)	24.89	28.88	40.44	41.30	29.99	21.34	23.48	22.9	24.99	72.50	90.36	38.5	5.71	69.50	59.77	59.50	40.88
							LLaMA-3-70B-Instruct, KV-size 128										
SnapKV	25.91	39.41	43.83	49.60	51.23	27.76	22.14	21.91	23.16	69.00	91.55	43.54	12.50	72.00	48.41	63.49	**44.09**
+THINK (0.4)	25.64	39.20	43.60	50.22	50.50	29.32	21.70	21.96	23.35	68.00	91.27	43.24	12.50	73.00	48.01	62.43	44.00
+THINK (0.5)	26.31	38.76	44.86	48.54	49.62	28.97	21.46	22.01	22.91	67.00	91.52	43.15	12.50	72.50	47.21	60.82	43.63
							LLaMA-3-70B-Instruct, KV-size 512										
SnapKV	27.95	45.19	48.50	50.97	54.53	29.78	25.34	22.36	26.03	73.50	92.63	45.07	12.50	72.50	45.21	68.22	46.27
+THINK (0.4)	27.47	45.31	48.57	51.22	54.32	30.05	25.42	22.72	26.20	73.50	92.13	45.53	12.50	73.00	48.32	66.99	**46.45**
+THINK (0.5)	26.97	44.55	48.16	50.84	53.80	30.57	25.29	22.65	25.53	73.00	92.13	43.66	12.50	73.00	50.52	64.82	46.12
							LLaMA-3-70B-Instruct, KV-size 1024										
SnapKV	26.80	46.21	49.93	51.70	54.71	29.86	27.61	22.43	27.15	73.50	92.38	46.18	12.50	72.50	42.84	69.89	46.64
+THINK (0.4)	27.04	46.01	50.13	51.96	54.36	29.87	27.74	22.78	27.07	73.50	91.88	46.35	12.50	73.00	45.05	67.87	**46.69**
+THINK (0.5)	27.62	46.22	48.97	51.79	53.39	30.47	27.45	23.05	26.57	73.50	91.88	43.99	12.50	72.50	47.41	66.84	46.51
							LLaMA-3-70B-Instruct, KV-size 2048										
SnapKV	27.44	46.51	49.60	51.80	54.77	31.05	29.67	22.44	27.43	73.50	92.38	45.98	12.50	72.50	41.86	68.72	**46.76**
+THINK (0.4)	27.13	46.26	50.04	51.72	55.03	31.19	29.75	22.47	27.28	73.50	91.88	46.37	12.50	72.50	42.66	67.77	46.75
+THINK (0.5)	27.84	46.86	49.18	51.97	53.58	31.44	29.41	22.89	27.33	73.50	91.88	43.60	12.50	72.50	44.78	66.65	46.62

4.2 Results on LongBench

Tables 2 and 3 present the results of KV compression methods and their integration with our proposed channel pruning technique for the key cache (THINK) across three different base LLMs, evaluated at various KV-sizes on the LongBench benchmark. The following observations can be drawn: (1) Our method successfully prunes the channels of the key cache after the KV cache has been compressed using H2O and SnapKV. For the LLaMA-3-8B-Instruct base model, our approach reduces memory usage while slightly improving performance for both H2O and SnapKV. For the Mistral-7B-Instruct-v0.2 base model, our method similarly reduces memory usage, with only a minor performance drop in some cases. For the larger LLaMA-3-70B base model, our method achieves comparable or superior performance after pruning 40% of the key cache channels, compared to the SnapKV baselines. (2) When Comparing SnapKV or H2O integrated with THINK in Table 2 to SnapKV or H2O integrated with l_1 or l_2 norm in Table 1, our query-driven channel pruning approach demonstrates superior performance when the pruning ratio of $\lambda = 0.4$. (3) Lower pruning ratios generally result in better performance compared to higher pruning ratios. (4) As the KV-size increases from 128 to 2048, the performance of our channel pruning method improves. Notably, with a KV-size of 2048 and

Table 3: Performance comparison of key cache pruning on Mistral-7B-Instruct-v0.2 on LongBench. THINK (λ) indicates we prune the key cache channels with a pruning ratio of λ.

Method	Single-Document QA			Multi-Document QA			Summarization			Few-shot Learning			Synthetic		Code		Avg.
	NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
Mistral-7B-Instruct-v0.2, KV-size Full																	
ALL KV	26.63	32.99	49.34	42.77	27.35	18.77	32.87	24.24	27.10	71.00	86.23	42.96	2.75	86.98	56.93	54.49	42.71
Mistral-7B-Instruct-v0.2, KV-size 128																	
H2O	21.21	21.81	38.87	30.42	20.36	12.30	20.58	22.61	22.10	39.00	82.37	40.44	2.90	79.56	51.22	48.38	34.63
+THINK (0.4)	21.17	21.90	39.29	29.92	20.99	12.30	20.84	22.91	21.92	39.00	82.70	40.35	2.97	79.21	51.19	48.32	**34.69**
+THINK (0.5)	21.67	21.80	39.48	28.74	20.65	13.14	20.57	22.83	21.78	39.00	82.54	40.12	3.61	78.39	50.27	48.4	34.56
+THINK (0.6)	21.04	21.30	39.56	28.68	21.29	13.97	20.13	22.52	21.81	39.50	82.05	39.14	4.16	74.23	49.83	47.67	34.18
SnapKV	19.17	21.40	42.93	36.76	22.44	15.86	19.16	21.84	21.55	47.50	84.15	40.24	2.30	68.26	52.31	48.80	**35.29**
+THINK (0.4)	20.52	21.00	42.65	37.58	22.09	15.23	19.29	22.01	21.22	47.00	83.85	39.64	3.20	67.45	51.48	48.31	35.16
+THINK (0.5)	20.67	20.60	43.37	37.27	21.58	15.66	19.06	21.79	21.02	47.00	83.38	39.77	3.65	67.06	50.80	48.35	35.06
+THINK (0.6)	21.25	20.82	44.20	36.21	21.68	16.47	19.05	21.99	20.73	45.00	83.81	38.79	4.19	66.90	49.99	47.61	34.92
Mistral-7B-Instruct-v0.2, KV-size 512																	
H2O	21.83	26.00	44.69	32.46	23.05	14.69	23.53	23.06	24.59	42.00	85.22	41.49	3.40	86.20	54.78	51.09	37.38
+THINK (0.4)	21.58	26.15	44.49	32.73	23.99	15.09	23.56	23.28	24.45	42.00	85.58	42.58	3.18	85.7	54.39	51.15	**37.49**
+THINK (0.5)	22.76	25.74	44.61	31.74	23.25	13.91	23.31	23.13	24.34	41.00	85.39	41.85	2.82	84.36	54.69	50.88	37.11
+THINK (0.6)	22.91	25.57	44.04	29.48	22.88	13.67	23.31	22.64	24.10	41.00	85.31	41.15	2.98	82.34	53.70	50.25	36.58
SnapKV	24.44	27.81	48.98	39.46	25.25	16.98	23.70	22.96	24.37	67.00	85.88	41.26	2.78	86.56	56.46	53.41	40.46
+THINK (0.4)	24.27	28.46	49.26	38.13	24.22	16.92	23.59	23.70	24.46	67.50	85.9	42.51	2.92	85.32	55.89	53.35	40.40
+THINK (0.5)	24.56	29.22	48.59	37.70	24.27	17.39	23.68	23.65	24.58	67.50	86.05	42.01	3.07	86.30	56.49	53.29	**40.52**
+THINK (0.6)	24.07	28.27	49.10	38.65	24.31	17.52	23.16	23.51	24.23	67.00	86.33	40.78	3.69	83.74	54.94	52.23	40.10
Mistral-7B-Instruct-v0.2, KV-size 1024																	
H2O	23.67	28.55	46.40	36.99	24.82	15.02	25.21	23.04	25.77	46.00	85.93	41.98	3.24	86.57	56.40	52.75	**38.90**
+THINK (0.4)	23.97	28.91	45.84	35.78	24.88	14.55	25.11	23.35	25.83	45.50	86.11	42.44	3.23	84.82	56.21	53.02	38.72
+THINK (0.5)	23.89	28.40	46.60	35.57	24.26	14.78	24.98	23.31	25.68	44.50	86.16	42.72	3.38	83.20	55.88	52.63	38.50
+THINK (0.6)	23.87	27.76	46.25	35.28	24.38	14.74	24.35	23.35	25.50	44.50	85.38	41.37	3.34	81.42	55.21	51.89	38.04
SnapKV	25.47	29.57	49.33	40.90	25.53	19.01	25.94	23.89	26.21	69.50	86.48	42.10	2.98	88.56	57.19	53.60	**41.64**
+THINK (0.4)	25.22	30.48	48.58	41.11	25.28	18.99	25.91	24.00	26.13	70.00	86.64	43.35	2.98	86.3	56.71	54.19	41.62
+THINK (0.5)	25.63	30.08	49.41	40.59	25.13	19.58	25.47	24.23	25.92	69.5	86.67	42.31	2.74	84.78	57.43	53.59	41.44
+THINK (0.6)	24.69	29.3	48.90	40.44	25.33	19.58	25.23	23.6	25.25	69.00	86.85	40.86	3.19	83.70	56.3	53.30	40.97
Mistral-7B-Instruct-v0.2, KV-size 2048																	
H2O	25.76	31.10	49.06	40.38	26.43	16.78	27.17	23.64	26.69	55.00	86.35	42.48	2.72	86.64	56.98	53.91	**40.69**
+THINK (0.4)	25.40	30.80	48.45	39.64	26.08	16.82	27.12	23.79	26.65	53.50	86.39	43.03	3.29	86.39	56.61	53.60	40.47
+THINK (0.5)	25.68	31.24	48.69	39.65	25.84	16.72	26.69	23.57	26.78	52.00	86.74	42.85	4.01	83.46	57.12	53.67	40.29
+THINK (0.6)	25.83	31.00	48.23	38.58	25.71	16.54	26.51	23.81	26.28	50.50	86.57	42.05	3.36	82.49	56.04	52.67	39.76
SnapKV	25.89	32.56	48.55	41.68	27.24	18.75	28.90	24.47	26.63	70.00	86.27	42.57	3.09	86.93	57.44	53.83	42.18
+THINK (0.4)	25.77	32.67	48.70	41.06	27.07	19.14	28.91	24.37	26.88	70.00	86.37	42.75	3.61	87.38	57.21	54.44	**42.27**
+THINK (0.5)	26.44	32.94	49.02	40.86	26.84	19.49	28.46	24.51	26.72	70.00	86.50	41.75	2.78	84.70	56.47	54.15	41.98
+THINK (0.6)	26.00	32.53	48.73	40.95	26.77	18.92	27.40	23.97	26.37	70.00	86.45	41.12	3.31	82.24	56.01	53.53	41.52

Table 4: Performance evaluation of combining THINK with KIVI [32] on LongBench. THINK (0.4) indicates we prune the key cache channels with a pruning ratio of $\lambda = 0.4$.

Method	Bit	Single-Document QA			Multi-Document QA			Summarization			Few-shot Learning			Synthetic		Code		Avg.
		NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
KIVI	2/2	19.47	18.62	30.28	29.42	25.00	10.30	21.34	20.51	25.10	63.00	85.04	40.16	4.00	8.00	58.04	52.48	31.92
+THINK (0.4)	2/2	19.46	19.01	30.52	28.79	25.78	9.53	22.11	20.66	25.73	63.00	84.62	41.54	3.50	7.00	56.51	48.92	31.77

a pruning ratio of 0.4, our method even surpasses the performance of LLaMA-3-8B-Instruct with a full KV cache. These findings suggest that our method is agnostic to the underlying KV cache compression techniques and can further enhance both performance and memory efficiency. Moreover, query-driven channel pruning proves more effective than l_1 and l_2 norm-based methods for channel pruning in LLMs. We further validate the efficacy of our method by applying it to the KV cache quantization technique KIVI [32], as shown in Table 4. First, we prune 40% of the key cache channels, followed by quantization of the remaining channels into 2-bit (implementation details provided in Appendix C.2). Compared to the standard KIVI method, our approach reduces KV cache memory by 20%, with minimal performance degradation.

4.3 Ablations

Performance Comparison Under the Same Memory Usage. To ensure a fair comparison, we adjust the KV-size of H2O or SnapKV to match the memory usage of H2O with THINK or SnapKV with THINK on Mistral-7B-Instruct-v0.2. For example, the KV-size of H2O with THINK is set to 128. Due to channel pruning applied to the key cache, the memory consumption of H2O with THINK at a KV-size of 128 is lower than that of H2O at the same KV-size. Consequently, the KV-size of

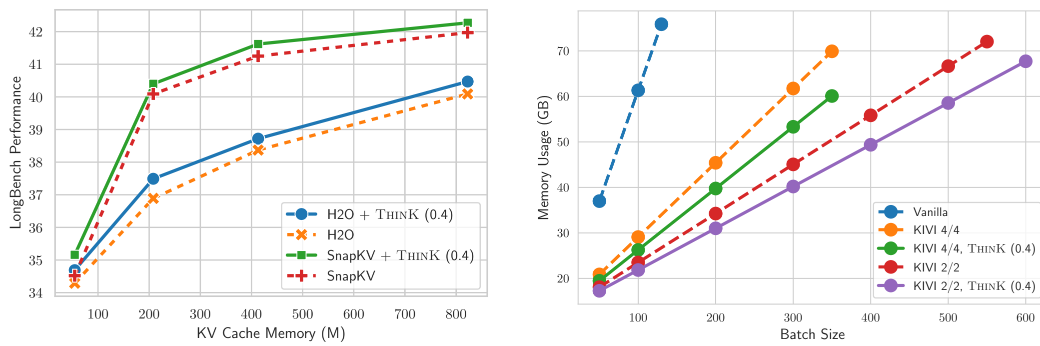

Figure 2: (a) presents the performance comparison with token eviction methods under identical memory usage for Mistral-7B-Instruct-v0.2, while (b) illustrates the memory usage comparison with the KV cache quantization method KIVI across different batch sizes for LLaMA-2-7B-chat. THINK (0.4) indicates we prune the key cache channels with a pruning ratio of $\lambda = 0.4$.

Table 5: Needles-in-a-Haystack Test Results

Model	Method	λ	KV-size			
			128	**512**	**1024**	**2048**
LLaMA3-8B-Instruct	SnapKV	0.0	79.6	90.2	91.2	91.7
	SnapKV+THINK	0.4	79.6	90.3	91.2	91.7
	SnapKV+THINK	0.5	77.4	89.6	91.0	91.7
Mistral-7B-Instruct-v0.2	SnapKV	0.0	77.8	89.5	90.4	90.8
	SnapKV+THINK	0.4	78.6	90.1	90.6	90.9
	SnapKV+THINK	0.5	78.1	90.1	90.8	91.1
	SnapKV+THINK	0.6	75.9	89.2	90.6	91.1

H2O is adjusted from 128 to 109 to equalize memory usage. Table 7 and Figure 2a present the results of these comparisons on the LongBench benchmark. The results demonstrate that H2O or SnapKV combined with THINK consistently outperforms their counterparts without THINK while maintaining the same memory footprint. This highlights the effectiveness of integrating query-driven channel pruning with KV cache compression methods, enabling more efficient memory utilization and improved compression of the KV cache.

Memory Usage Comparison. To evaluate the efficiency of THINK, we follow the methodology used in KIVI [32]. We generate synthetic workloads with an input prompt length of 160 and an output length of 338. The peak memory usage is reported for the vanilla FP16 baseline, KIVI, and KIVI combined with THINK (0.4) for LLaMA-2-7B-chat. As in Figure 2b, the memory savings from our method become increasingly evident as the batch size grows, in both the KIVI $2/2$ and KIVI $4/4$ configurations. Compared to the baseline model, our approach achieves over a $5\times$ increase in batch size while maintaining the same memory footprint when integrated with KIVI. More ablations are presented in Appendix A.2.

4.4 Results on Needle-in-a-Haystack

Table 5 presents the results of the Needle-in-a-Haystack test, using the SnapKV [28] approach with varying KV-sizes, ranging from 128 to 2048. With a modest pruning ratio of $\lambda = 0.4$, THINK consistently outperforms or matches the accuracy of the original SnapKV across both LLaMA-3 and Mistral models, regardless of KV-size. These comparisons demonstrate that the proposed query-driven channel pruning method effectively retains informative channels while discarding noisy ones. However, when the pruning ratio increases to $\lambda \geq 0.5$, we observe a drop in accuracy with smaller KV-sizes, particularly for 128 and 512, across both LLaMA-3 and Mistral models. Despite this, THINK achieves comparable performance with SnapKV when the KV-size is larger(*i.e.*, 1024 and

2048). Intuitively, a larger pruning ratio with a smaller KV-size may lead to the loss of more critical information compared to scenarios with a larger KV-size. In addition, the performance on larger KV-sizes suggests that THINK is robust for long-context tasks.

Figure 5 (a)-(d) (in Appendix B) visualize the Needle-in-a-Haystack accuracy across varying token lengths and depth percentages. The KV-sizes are set to 128 and 1024, with pruning ratios of $\lambda = 0.4$ and $\lambda = 0.5$, respectively. THINK preserves the retrieval capabilities of SnapKV, although there are minor numerical differences in overall accuracy (e.g., 77.8 vs. 78.6 and 90.4 vs. 90.6). THINK matches SnapKV in accuracy for the majority of token limits and depths, demonstrating consistency in performance. Furthermore, THINK successfully retrieves certain "needles" that SnapKV fails to capture, resulting in improved overall accuracy. These visualizations highlight the robustness of THINK from a fine-grained perspective, illustrating its capacity to enhance the original approach.

5 Related Work

In scenarios involving long contexts, the most significant computational burden from the attention mechanism is the key-value (KV) cache. Reducing the KV cache is a high priority for optimizing deployment efficiency. System-level optimizations, such as FlashAttention [8] and PagedAttention [26], have been developed to address this issue. Additionally, algorithm-level optimizations are being explored to further enhance efficiency.

KV Cache Eviction. StreamingLLM [51] maintains a few initial tokens and some recent tokens based on the observation of attention sink, which may result in the loss of important information carried by the dropped tokens. H2O [58] retains only a small portion of the tokens by greedily dropping tokens based on their contributions to the cumulative attention. SnapKV [28] selects clustered important KV positions for each attention head from an 'observation' window located at the end of the prompts. FastGen [16] adaptively evicts tokens from attention heads that emphasize local contexts. This approach focuses on discarding non-special tokens centered on special tokens, while the standard KV cache is used only for attention heads that broadly attend to all tokens. PyramidKV [57] and PyramidInfer [54] considers adjusting the KV cache size across different layers by allocating more cache in the lower layers and less in the higher ones.

KV Cache Quantization. SmoothQuant [50] can quantize the KV cache to 8-bit with minimal performance degradation. Q-Hitter [58] uses accumulated attention scores and 'Quantization Friendliness' metrics to identify tokens that are essential for maintaining the generalization capabilities of LLMs and are suitable for KV cache quantization. Some studies have found that the key cache and value cache should be quantized differently [32, 20]: the key cache should be quantized per-channel, while the value cache should be quantized per-token.

Structured Pruning of LLMs. Structured pruning [33, 11] of LLMs traditionally focuses on removing unimportant layers, heads, and hidden dimensions, which often results in significant performance degradation. In contrast, our methodology preserves the original architecture of the LLM and specifically targets the channel dimension within each head's key cache. By dynamically identifying unimportant channels based on data dependant criterion, our approach greatly reduce the key cache-size with negligible performance loss.

6 Conclusion

Motivated by the observation that certain channels have significantly larger magnitudes compared to others, and the singular value analysis indicates that the key cache is inherently low-rank [39], we propose THINK to perform pruning over the key cache channel. The proposed pruning strategy is query-dependant and optimized based on the attention scores, ensuring that essential information is retained for each input query. In addition, THINK can be seamlessly integrated with other popular token-level KV cache quantization techniques [28, 58], further enhancing inference efficiency. Extensive experiments on LongBench [2] and Needle-in-a-Haystack tests with two foundation models demonstrate the effectiveness and robustness of our query-dependent channel pruning method. Our approach achieves comparable or superior performance to baseline methods while reducing the key cache size by 40%. Our analysis indicates that THINK can maintain superior performance over baselines with a smaller KV cache size under equivalent memory consumption conditions.

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A Appendix

A.1 Observations

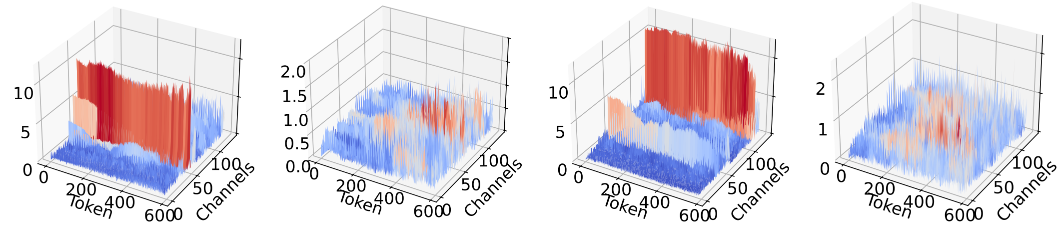
Magnitudes of KV cache channels. Figure 3 visualizes the absolute values of the KV cache across tokens in each channel[1]. Consistent with previous findings [30, 50, 32], we observe that only certain channels have significant magnitudes in the key cache, whereas the value cache lacks obvious patterns. For instance, in layer 14 (Figure 3 (a)), the magnitudes in the key cache are substantially higher around the 50^{th} channel across all tokens. A similar pattern is observed in the 50^{th} and 150^{th} channels of the first head in layer 20 (Figure 3 (c)). Given such an observation, [32] proposed to perform quantization over the channels of the key cache. Beyond quantization, our findings suggest that certain key cache channels with smaller contributions to the attention mechanism can be pruned. Moreover, channel quantization and pruning are orthogonal techniques, meaning they can be applied concurrently to further improve model efficiency.

Singular value analysis. We conducted singular value decomposition (SVD) [9] on the attention weights of the specified layers to investigate their principal components. The singular values derived from SVD capture the effective rank of the attention matrix, indicating how the information is distributed across different components.

Figure 4 (a) illustrates the energy distribution of the singular values, plotted on a logarithmic scale to enhance visibility of the differences. Notably, only a few singular values exhibit high energy levels exceeding 0.01 across all heads and layers, highlighting their relative significance. This observation aligns with previous findings [3], where a small subset of singular values often captures most of the information in attention mechanisms. In addition, the rapid decay of the energy suggests that a low-rank approximation can effectively capture the essential information in the key cache.

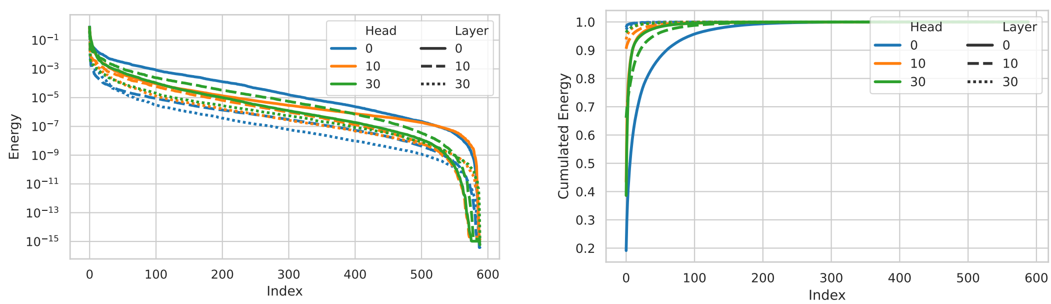
Figure 4 (b), the normalized cumulative energy sum reveals that the top 50 singular values account for over 90% of the total energy. These findings suggest that the attention matrix is inherently low-rank [47, 6, 13], indicating that the key cache can be approximated using low-dimensional vectors [39].

[1] We use the visualization code from `https://github.com/jy-yuan/KIVI/tree/main/vis`.



(a) Key Cache (14^{th} layer) (b) Value Cache (14^{th} layer) (c) Key Cache (20^{th} layer) (d) Value Cache (20^{th} layer)

Figure 3: Magnitude of key and value cache for LLaMA3-8B. The first head of layer 14 and layer 20 of LLaMA3-8B is selected to visualize the magnitude of the key and value caches. We observe that the magnitudes of the key cache channels vary differently, whereas the channels of the value cache do not exhibit such variation.



(a) Energy of singular values (b) Cumulative energy of singular values

Figure 4: The energy and cumulative energy of the singular values.

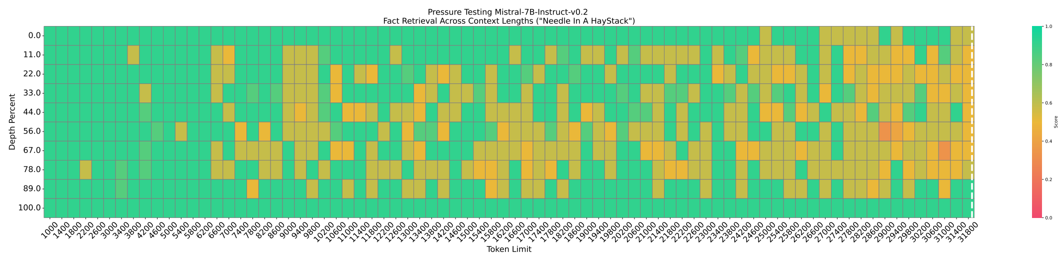
A.2 Ablation Studies

Impact of Different Recent Sizes. Preserve the most recent KV embeddings [58, 28] is important for maintaining the performance of LLMs after KV cache compression. Note that a tradeoff exists: increasing the recent-size allows more infomation to be propagated, while increasing the cache size to be stored. To study its impacts, we evaluate the performance produced by three recent-size, namely 0, 32 and 128, on LongBench, using Mistral-7B-Instruct-v0.2 as the basleline model. The results are summarized in Table 6. One can observe that a recent-size of 32 yields superior performance than 0 in terms of averaged score on LongBench which demonstrates the importance of keeping the most recent KVs. On the other hand, the performance of 32 and 128 is negligible, suggesting that maintaining the most recent 32 KVs suffices to preserve optimal performance.

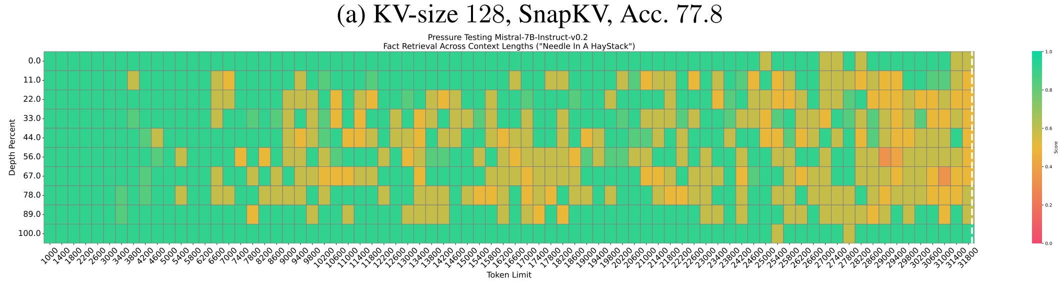
Pruning Channels of Both Key and Value Cache. In our previous study, we evaluate methods with pruning channels of the key cache. In this part, we explore the impact of pruning channels of the value cache. Specifically, for KV cache compression methods, we prune channels of the key cache at one pruning ratio and channels of the value cache at another pruning ratio. Table 8 presents the results with LLaMA3-8B and Mistral-7B on LongBench. When testing on the base model LLaMA3-8B, H2O or SnapKV with key and value channel pruning can perform on par with H2O or SnapKV without channel pruning. In some cases, H2O or SnapKV with key and value channel pruning may even perform better. Pruning the channels of the value cache over the base model Mistral-7B experiences a slight performance drop. Despite this, note that KV cache with additional value channel pruning can further reduce memory usage.

B Needle-in-a-Haystack test performance comparison

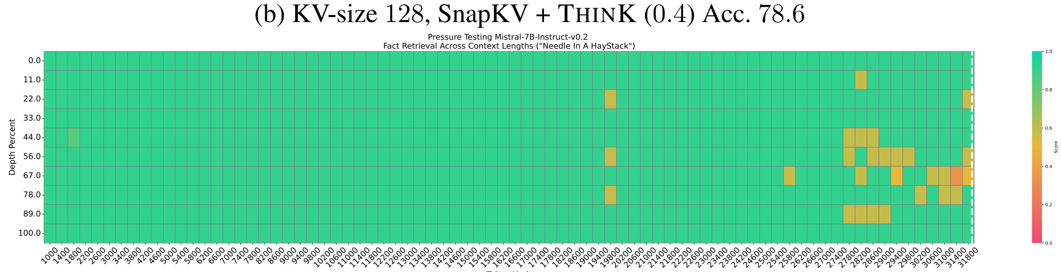
Figure 5 visualizes the test performance comparison on Needle-in-a-Haystack on Mistral-7B-Instruct-v0.2.



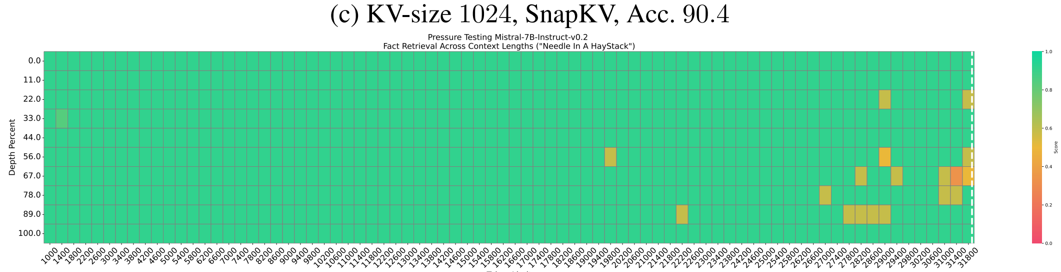
(a) KV-size 128, SnapKV, Acc. 77.8



(b) KV-size 128, SnapKV + THINK (0.4) Acc. 78.6



(c) KV-size 1024, SnapKV, Acc. 90.4



(d) KV-size 1024, SnapKV + THINK (0.5), Acc. 90.8

Figure 5: Needle-in-a-Haystack test performance comparison with Mistral-7B-Instruct-v0.2. THINK (λ) indicates we prune the key cache channels with a pruning ratio of λ

Table 6: Performance comparison of key cache pruning with varying recent-sizes.

Recent-Size	Single-Document QA			Multi-Document QA			Summarization			Few-shot Learning			Synthetic		Code		Avg.
	NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
H2O + THINK ($\lambda = 0.4$)																	
0	24.40	27.50	45.42	35.17	24.45	13.02	27.65	23.88	26.86	53.50	86.06	41.73	3.01	83.42	55.12	51.32	38.91
32	25.40	30.80	48.45	39.64	26.08	16.82	27.12	23.79	26.65	53.50	86.39	43.03	3.29	86.39	56.61	53.60	40.47
128	25.69	30.93	48.32	39.63	26.08	16.82	27.18	23.92	26.62	53.50	86.39	42.96	3.29	86.39	56.77	53.60	**40.51**
SnapKV + THINK ($\lambda = 0.4$)																	
0	24.94	28.58	45.78	39.59	25.40	15.92	29.50	24.05	26.72	70.00	85.60	41.38	2.97	84.00	55.27	52.39	40.76
32	25.77	32.67	48.70	41.06	27.07	19.14	28.91	24.37	26.88	70.00	86.37	42.75	3.61	87.38	57.21	54.44	**42.27**
128	25.75	32.49	48.61	41.01	27.18	19.14	28.79	24.64	26.77	70.00	86.37	42.77	3.61	87.13	57.19	54.39	42.24

Table 7: Performance comparison of key cache pruning with the same memory consumption.

Methods	Memory(M)	Single-Document QA			Multi-Document QA			Summarization			Few-shot Learning			Synthetic		Code		Avg.
		NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
H2O																		
Vanilla	54.5	21.29	20.69	37.66	28.65	21.08	14.01	20.20	22.11	21.33	38.50	82.55	39.87	3.66	78.14	50.32	48.54	34.29
THINK	54.4	21.17	21.90	39.29	29.92	20.99	12.30	20.84	22.91	21.92	39.00	82.70	40.35	2.97	79.21	51.19	48.32	**34.69**
Vanilla	208.0	22.13	23.83	43.24	30.92	23.36	14.56	22.92	22.77	24.23	41.50	85.04	41.26	3.02	86.03	54.91	50.50	36.89
THINK	208.0	21.58	26.15	44.49	32.73	23.99	15.09	23.56	23.28	24.45	42.00	85.58	42.58	3.18	85.7	54.39	51.15	**37.49**
Vanilla	413.0	22.90	28.45	46.16	35.57	23.86	13.74	24.90	23.19	25.77	44.50	85.54	41.97	3.22	85.82	55.96	52.33	38.37
THINK	412.8	23.97	28.91	45.84	35.78	24.88	14.55	25.11	23.35	25.83	45.50	86.11	42.44	3.23	84.82	56.21	53.02	**38.72**
Vanilla	822.5	25.51	30.23	48.23	39.72	25.56	16.75	26.98	23.81	26.47	50.50	86.43	42.09	2.78	85.57	57.4	53.42	40.09
THINK	822.4	25.40	30.80	48.45	39.64	26.08	16.82	27.12	23.79	26.65	53.50	86.39	43.03	3.29	86.39	56.61	53.60	**40.47**
SnapKV																		
Vanilla	54.5	19.25	19.95	42.80	35.88	21.96	14.59	18.76	21.71	20.70	46.00	84.12	39.43	2.59	65.36	51.39	47.81	34.52
THINK	54.4	20.52	21.00	42.65	37.58	22.09	15.23	19.29	22.01	21.22	47.00	83.85	39.64	3.20	67.45	51.48	48.31	**35.16**
Vanilla	208.0	23.31	27.45	48.85	38.77	23.93	16.50	23.44	23.63	24.13	66.00	86.05	41.00	2.62	87.01	56.13	52.60	40.09
THINK	208.0	24.27	28.46	49.26	38.13	24.22	16.92	23.59	23.70	24.46	67.50	85.90	42.51	2.92	85.32	55.89	53.35	**40.40**
Vanilla	413.0	24.24	29.53	49.13	40.48	25.05	18.74	25.46	23.64	25.60	68.00	86.14	41.42	3.03	88.55	57.08	53.86	41.25
THINK	412.8	25.22	30.48	48.58	41.11	25.28	18.99	25.91	24.00	26.13	70.00	86.64	43.35	2.98	86.30	56.71	54.19	**41.62**
Vanilla	822.5	24.84	31.90	48.16	41.32	26.77	19.49	28.23	24.63	26.41	70.00	86.32	41.83	2.91	88.06	56.98	53.74	41.97
THINK	822.4	25.77	32.67	48.7	41.06	27.07	19.14	28.91	24.37	26.88	70.00	86.37	42.75	3.61	87.38	57.21	54.44	**42.27**

C Implementations

C.1 Implementation of THINK

Following SnapKV, we focus on the long context input scenario. We opt not to prune the most recent tokens and newly generated keys. Consequently, our key-value (KV) cache will store two distinct categories of keys: one subset consists of pruned keys with a reduced channel size, while the other retains keys at their original size. Additionally, we maintain a binary mask to indicate which channels have been pruned. Note that the memory overhead associated with this mask is negligible. Figure 6 illustrates one implementation of our method during the decoding stage. This implementation involves initially pruning the query using the mask. The pruned query is then multiplied by the pruned key, while the unpruned query is applied to the unpruned Key. Subsequently, the two outputs are concatenated.

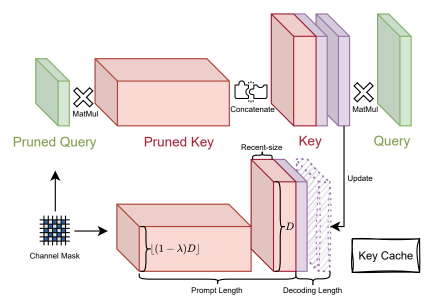


Figure 6: Implementation during decoding.

C.2 Implementation with quantization

Figure 7 illustrates the implementation of our method when integrated with the KV cache quantization method KIVI [32]. During the prefill phase, we first prune the unimportant channels of X_K before applying quantization. In the decoding phase, each newly arrived key cache t_K is added to X_{K_r}. Once X_{K_r} reaches G tokens, the residual length hyperparameter in KIVI, we prune and quantize the data, then concatenate it with the previously quantized $Q(P(X_{K_g}))$.

Table 8: Performance comparison of pruning both K and V cache with different pruning ratios on LongBench. H2O + THINKV (λ_1+λ_2) indicates that the key cache channels of H2O are pruned with a pruning ratio of λ_1 and the value cache channels are pruned of a pruning ratio of λ_2.

	Method	Single-Document QA			Multi-Document QA			Summarization			Few-shot Learning			Synthetic		Code		Avg.
		NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
LLaMA-3-8B-Instruct	KV-size 128																	
	H2O	22.12	13.20	31.61	37.79	32.71	18.45	20.32	22.02	21.10	38.50	87.75	39.14	5.83	69.50	55.06	50.97	35.38
	+THINKV (0.3+0.3)	23.71	13.65	33.08	41.86	29.88	18.04	19.60	21.65	20.26	38.00	86.08	38.61	5.16	69.50	57.59	55.19	**35.74**
	SnapKV	21.19	13.55	32.64	38.75	29.64	18.73	18.98	21.62	20.26	45.00	88.36	37.64	5.13	68.85	55.84	51.82	35.50
	+THINK(0.3+0.3)	21.86	13.79	33.26	40.93	29.39	19.22	18.81	21.30	19.26	41.50	87.00	37.95	5.78	69.50	57.84	55.62	**35.81**
	KV-size 512																	
	H2O	23.52	17.93	34.68	42.11	33.52	19.92	22.11	22.56	23.82	41.00	90.46	40.20	5.87	69.50	56.71	51.69	**37.23**
	+THINKV (0.3+0.3)	22.83	17.57	34.18	42.67	33.52	19.95	21.17	22.23	22.82	38.50	90.11	39.08	5.21	69.0	59.99	56.83	**37.23**
	SnapKV	24.84	23.96	38.77	42.75	34.55	20.87	22.26	22.61	23.97	70.00	90.52	40.29	5.81	69.50	59.04	51.81	40.10
	+THINKV (0.3+0.3)	24.57	24.59	38.09	44.61	34.37	20.37	21.23	21.95	23.30	66.00	90.69	39.38	5.60	69.00	61.75	58.46	**40.25**
	KV-size 2048																	
	H2O	25.56	26.85	39.54	44.30	32.92	21.09	24.68	23.01	26.16	53.00	90.65	41.84	4.91	69.25	58.43	51.31	**39.59**
	+THINKV (0.3+0.3)	25.03	26.77	39.68	42.12	33.08	19.59	23.00	22.89	25.27	51.00	91.11	40.58	5.23	69.00	61.12	57.95	**39.59**
	+THINKV (0.4+0.4)	24.87	24.31	37.77	43.13	34.42	19.60	21.67	22.70	24.52	49.00	90.81	39.28	6.00	69.00	61.81	58.08	39.19
	SnapKV	25.86	29.55	41.10	44.99	35.80	21.81	25.98	23.40	26.46	73.50	90.56	41.66	5.17	69.25	58.67	51.52	**41.58**
	+THINKV (0.3+0.3)	25.13	29.97	40.35	44.12	34.64	19.94	23.62	23.03	25.30	72.50	90.78	39.46	5.35	69.00	61.50	57.91	41.41
	+THINKV (0.4+0.4)	25.13	28.85	40.70	44.21	36.36	21.07	22.31	22.89	24.80	72.50	90.88	38.77	6.41	69.00	61.49	58.87	41.52
Mistral-7B-Instruct-v0.2	KV-size 128																	
	H2O	21.21	21.81	38.87	30.42	20.36	12.30	20.58	22.61	22.10	39.00	82.37	40.44	2.90	79.56	51.22	48.38	**34.63**
	+THINKV (0.3+0.3)	20.71	21.49	38.01	30.66	22.28	13.87	20.13	22.45	21.07	38.50	82.20	38.69	2.94	78.56	51.55	48.28	34.46
	SnapKV	19.17	21.40	42.93	36.76	22.44	15.86	19.16	21.84	21.55	47.50	84.15	40.24	2.30	68.26	52.31	48.80	**35.29**
	+THINKV (0.3+0.3)	19.92	20.61	42.68	37.63	23.19	15.09	18.97	21.93	20.55	45.00	84.06	39.33	2.99	66.00	51.51	47.51	34.81
	KV-size 512																	
	H2O	21.83	26.00	44.69	32.46	23.05	14.69	23.53	23.06	24.59	42.00	85.22	41.49	3.40	86.20	54.78	51.09	**37.38**
	+THINKV (0.3+0.3)	22.36	24.26	44.77	30.47	22.94	14.96	22.63	22.90	23.73	41.50	85.30	40.21	3.08	80.07	54.48	50.96	36.54
	+THINKV (0.3+0.1)	22.14	25.15	45.29	31.78	23.21	14.62	23.36	22.70	24.51	41.50	85.61	41.58	2.75	84.03	54.50	51.09	37.11
	SnapKV	24.44	27.81	48.98	39.46	25.25	16.98	23.70	22.96	24.37	67.0	85.88	41.26	2.78	86.56	56.46	53.41	**40.46**
	+THINKV (0.3+0.3)	24.10	27.04	47.76	38.66	25.45	17.51	22.64	22.81	23.91	66.00	86.62	39.91	3.36	82.24	55.96	52.81	39.80
	+THINKV (0.3+0.1)	23.90	28.14	48.35	39.03	24.83	16.68	23.51	23.12	24.34	67.50	86.09	41.69	2.65	84.34	57.29	53.22	40.29
	KV-size 1024																	
	H2O	23.67	28.55	46.4	36.99	24.82	15.02	25.21	23.04	25.77	46.00	85.93	41.98	3.24	86.57	56.40	52.75	**38.90**
	+THINKV (0.3+0.3)	23.65	26.54	47.00	35.52	24.79	17.15	23.64	23.12	25.20	44.00	86.38	41.67	3.46	80.14	56.53	52.86	38.23
	+THINKV (0.3+0.1)	24.13	28.57	46.31	35.59	24.92	15.34	24.58	23.33	25.93	45.50	85.91	42.97	2.57	83.64	55.39	52.73	38.59
	SnapKV	25.47	29.57	49.33	40.90	25.53	19.01	25.94	23.89	26.21	69.50	86.48	42.10	2.98	88.56	57.19	53.60	**41.64**
	+THINKV (0.3+0.3)	25.29	29.25	49.17	41.25	25.75	19.37	24.64	23.02	25.27	69.00	86.70	40.92	3.29	82.06	57.15	54.15	41.02
	+THINKV (0.3+0.1)	25.84	29.30	49.56	41.44	25.29	19.02	25.21	23.73	25.72	69.00	86.69	42.55	2.44	85.76	57.55	54.10	41.45
	KV-size 2048																	
	H2O	25.76	31.10	49.06	40.38	26.43	16.78	27.17	23.64	26.69	55.0	86.35	42.48	2.72	86.64	56.98	53.91	**40.69**
	+THINKV (0.3+0.3)	25.60	28.74	47.54	38.67	26.25	17.35	24.54	23.27	26.15	51.00	87.01	43.02	2.94	81.46	56.41	54.26	39.64
	+THINKV (0.3+0.1)	25.64	30.65	48.95	40.42	26.43	16.65	26.76	23.51	26.59	52.50	86.53	43.45	2.66	83.96	56.55	53.83	40.32
	SnapKV	25.89	32.56	48.55	41.68	27.24	18.75	28.90	24.47	26.63	70.00	86.27	42.57	3.09	86.93	57.44	53.83	**42.18**
	+THINKV (0.3+0.3)	27.01	30.72	48.81	41.15	26.93	18.93	25.81	23.59	26.42	70.00	86.82	41.91	3.05	82.65	57.01	54.25	41.57
	+THINKV (0.3+0.1)	26.22	32.69	48.96	40.83	26.70	19.02	27.87	24.23	26.64	70.00	86.65	42.63	2.22	85.13	57.00	54.28	41.94

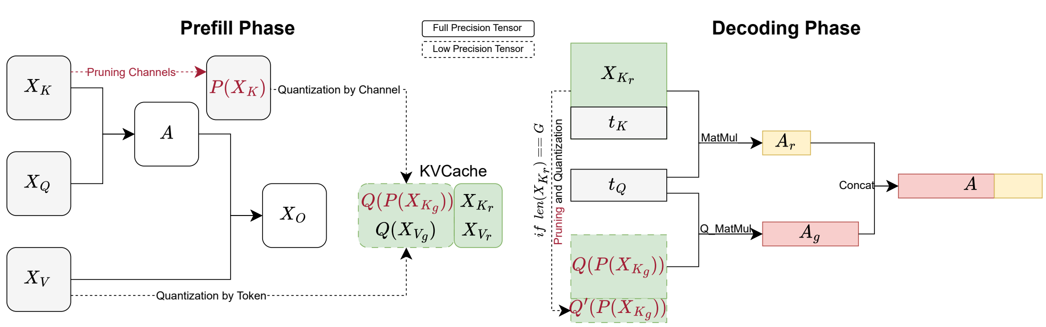


Figure 7: Implementations of THINK when incorporated with KIVI.